Oligopoly

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ECON 2216: Industrial Organization

Outline

1. Oligopoly
   - Assumptions
   - Terminology
   - Single-Period Oligopoly Models
   - Multiperiod Games
   - Mathematical Derivation - Cournot and Stackelberg Model

2. Product Differentiation and Monopolistic Competition
   - Differentiated Products
   - Location Model

Outline

Outline

Assumptions

- **Oligopoly**: a small number of firms acting independently but aware of one another's existence. Unlike monopolistic and competitive firms, oligopolists cannot ignore other firms' actions

1. Consumers are price takers
2. All firms produce homogeneous products
3. There is no entry into the industry, so the number of firms remains constant over time
4. Firms collectively have market power: They can set price above marginal cost
5. Each firm only sets its price or output (not advertising or other variables)

Outline

Terminology

- Strategic decision makers: **players** (e.g. firms)
- A **game** is any competition in which strategic behavior is important
- Each firm forms a **strategy** it will take to compete with other firms
- Each firm's **payoff** depends on the actions of all the firms

Outline

Nash Equilibrium

- Nash equilibrium: no firm can obtain a higher payoff by choosing a different strategy
  - In the Cournot and Stackelberg models, firms' strategies concern setting quantities
  - In the Bertrand model, firms set prices

Cournot Model[1]

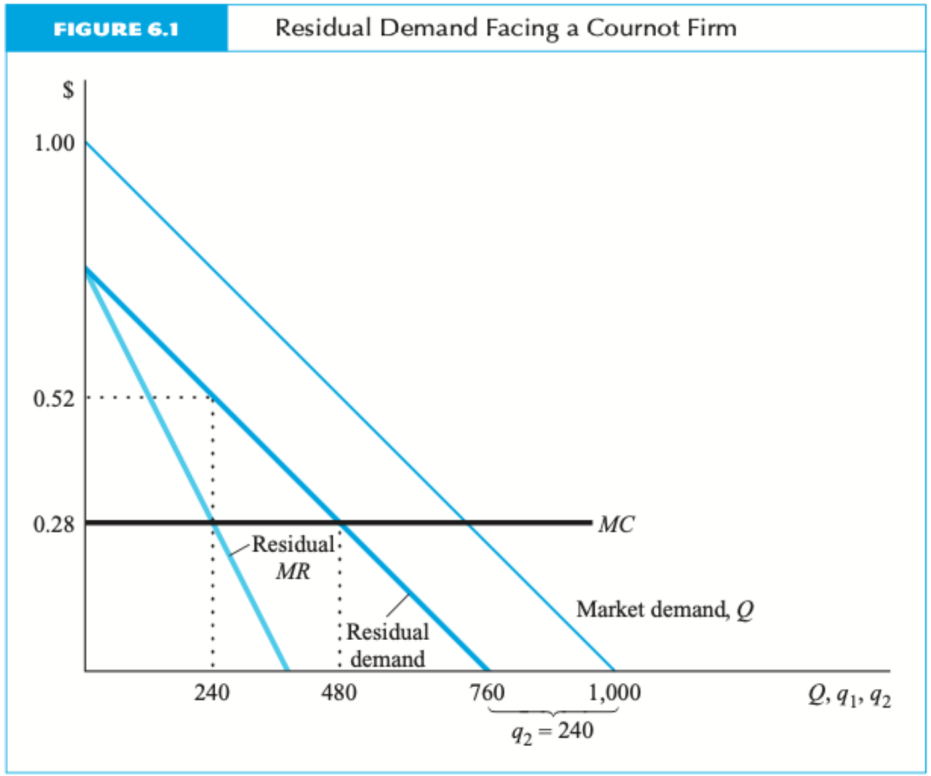
- Objective
  - each firm acts independently and attempts to maximize its profits by choosing its output

- Assumptions
  - No entry: There are two firms and no entry by other firms is possible
  - Homogeneity: The firms produce products
  - Single period: This market and the two firms only exist for one period
  - Demand: The market demand curve is a linear function of price
  - Costs: Each firm has a constant marginal cost

Cournot Model[2]

- Mechanism
  - Firm 1 chooses its output level depends on its belief about Firm 2's behavior:
    - ★ sell all but q_2 units of the amount demanded by the market - it faces the residual demand curve

$$q_1 = Q(p) = q_2$$



Cournot Model[3]

- Mechanism (cont'd)
  - Firm 1 has a monopoly over those consumers whose demands are not met by Firm 2
  - To maximize its profit, firm 1 sets q_1 where its derived residual demand curve intersects its marginal cost curve
    - ⋆ a **best-response function** (or **reaction function**): shows the highest profit action by a firm given its beliefs about the action its rival takes
    - ⋆ Firm 1's best-response function is

$$q_1 = R_1(q_2) = 360 - \frac{q_2}{2}$$

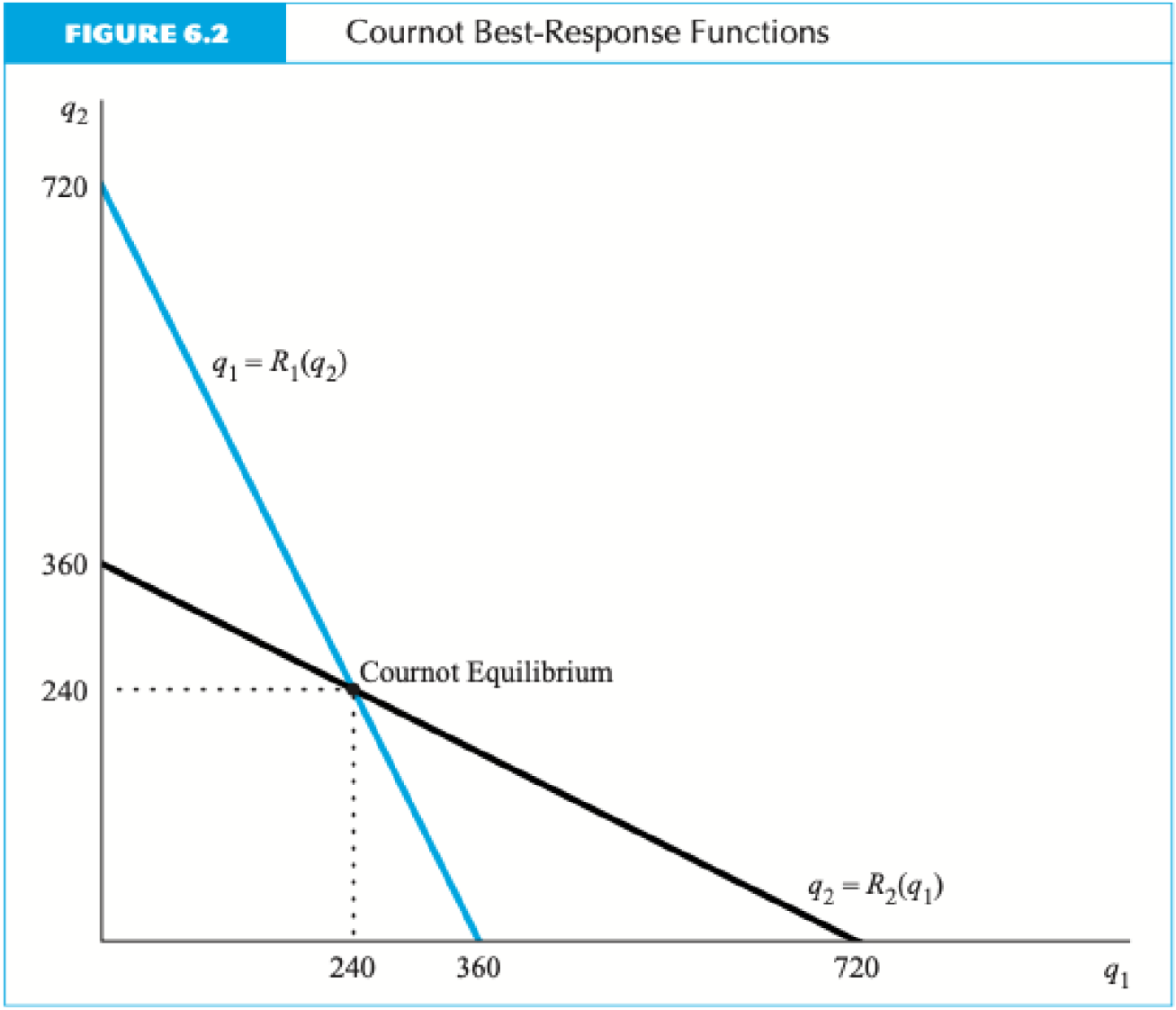
  - Firm 2's best-response function is the mirror image of Firm 1's

$$q_2 = R_2(q_1) = 360 - \frac{q_1}{2}$$

Cournot Model[4]

- Mechanism (cont'd)
  - **Equilibrium**:
    - ★ two firms' best-response functions cross once at $q_1 = q_2 = 240$
    - ★ Point of intersection $(240, 240)$ of the best-response functions: **Cournot equilibrium**
    - ★ In the Cournot equilibrium, each firm sells the quantity that maximizes its profits given its beliefs about the best response to the other firm's output level
    - ★ A firm is unwilling to produce at a point not on its best-response function because doing so would result in a lower profit

Cournot Model[5]



Cournot Model[6]

- A Comparison of the Cournot and Cartel Equilibria

TABLE 6.1 **A Comparison of Oligopoly Equilibria: A Linear Demand and Constant Marginal Cost Example**

	Output			Profits ($)		
	Firm	Industry	Price (¢)	Firm	Industry	Consumer Surplus
Monopoly	360	360	64	129.60	129.60	64.8
Cournot Duopoly	240	480	52	57.60	115.20	115.2
Stackelberg Duopoly		540	46		97.20	145.8
Leader	360			64.8		
Follower	180			32.4		
Competition*		720	28	0	0	259.2
Cournot: n firms	$\frac{720}{n+1}$	$\frac{720n}{n+1}$	$\frac{100+28n}{n}$	$\frac{518.4}{(n+1)^2}$	$\frac{518.4n}{(n+1)^2}$	$\frac{259.2n^2}{(n+1)^2}$
Stackelberg: n firms		$\frac{360(2n-1)}{n}$	$\frac{28n+36}{n}$		$\frac{129.6(2n-1)}{n^2}$	$\frac{64.8(2n-1)^2}{n^2}$
Leader	360			$\frac{129.6}{n}$		
Followers	$\frac{360(n-1)}{n}$			$\frac{129.6(n-1)}{n^2}$		

Market demand: $Q = 1{,}000 - 1{,}000\,p$
$MC = 28¢$
*Efficient point, Bertrand equilibrium, Cournot equilibrium with unlimited number of firms.

Cournot Model[7]

- A Comparison of the Cournot and Cartel Equilibria (cont'd)
  - The firms are worse off and the consumers are better off at the Cournot equilibrium than if firms act collusively as a cartel
  - At the social optimum output, consumer surplus is greater than under a cartel and a Cournot duopoly
  - The Cournot duopoly equilibrium lies between the competitive and monopolistic equilibria

Cournot Model[8]

- Three or More Cournot Firms

TABLE 6.2 Cournot Equilibrium with Few and Many Firms

			Firm		Industry	
	Number of Firms	Price (¢)	Output	Profit ($)	Output	Profits ($)
Monopoly	1	64	360	129.60	360	129.60
	2	52	240	57.60	480	115.20
	3	46	180	32.40	540	97.20
	4	42.4	144	20.74	576	82.94
	5	40	120	14.40	600	72.00
	6	38.3	102.9	10.58	617.1	63.48
	7	37	90	8.10	630	56.70
	8	36	80	6.40	640	51.20
	9	35.2	72	5.18	648	46.66
	10	34.5	65.5	4.28	654.5	42.84
	15	32.5	48	2.30	675	32.26
	20	31.4	34.3	1.18	685.7	23.51
	50	29.4	14.1	0.20	705.9	9.97
	100	28.7	7.1	0.05	712.9	5.08
	500	28.1	1.4	0.002	718.6	1.03
	1000	28.1	0.7	0.001	719.3	0.52
Competition	∞	28	~0	0.00	720	0.00

Cournot Model[9]

- Three or More Cournot Firms (cont'd)
  - If there are $n (\geq 2)$ identical Cournot firms, the same type of analysis can be used to derive the Cournot equilibrium
  - The larger is n, the smaller is output per firm, whereas the larger is industry output, and the lower is price
  - If the number of firms is extremely large, the output per firm, industry price, and industry output approach the socially optimal levels
    - ★ Consumers are better off (lower prices, higher consumer surplus) and firms are worse off (lower profits) as the number of firms increases

Betrand Model[1]

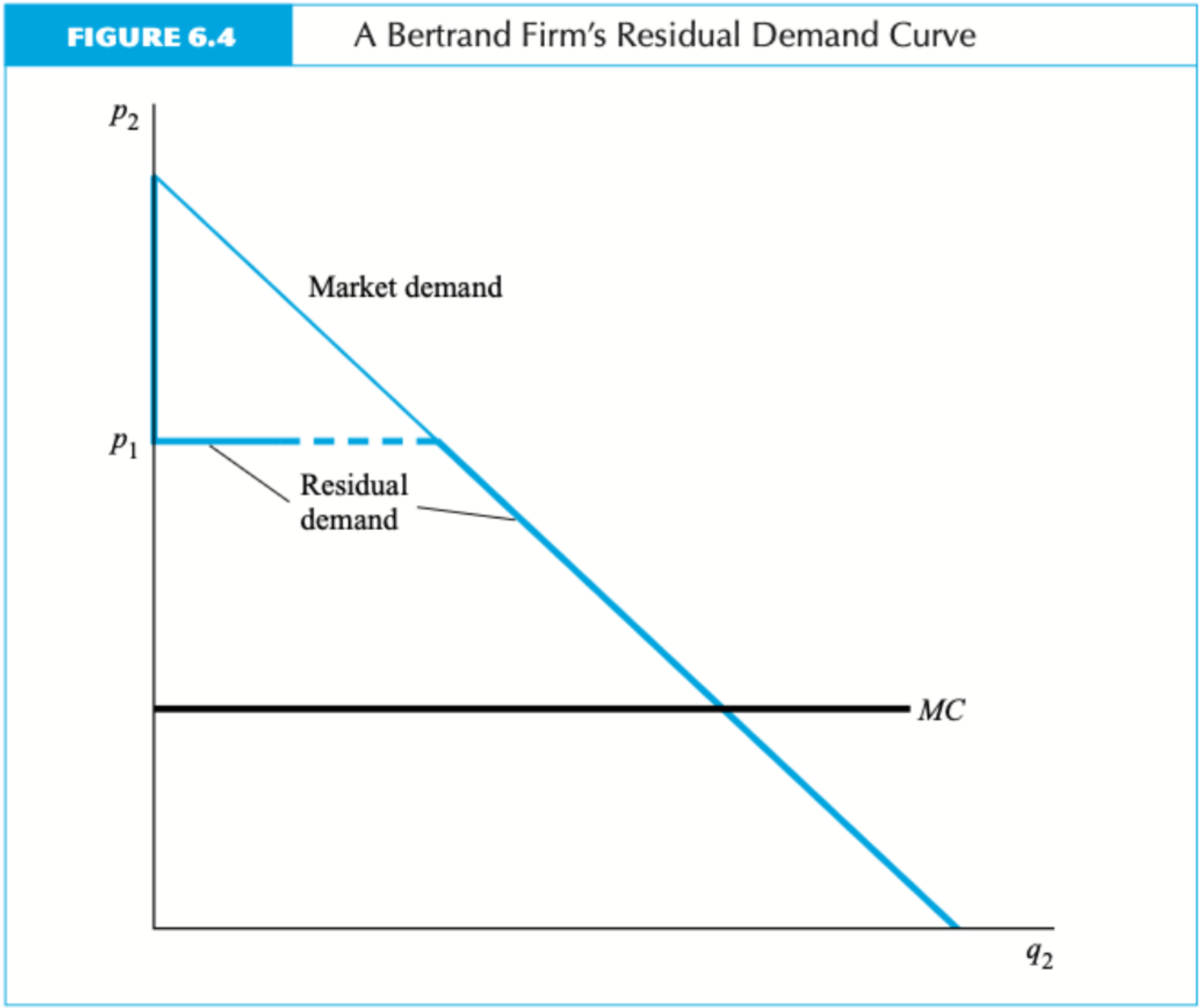
- Objective
  - Each firm acts independently and attempts to maximize its profits by choosing its prices rather than output:
    - ⋆ each firm believes its rival's price is fixed; by a slight price cut, the firm is able to capture all its rival's business
    - ⋆ firms make zero profits and no firm can increase its profits by raising or lowering its price, which is equivalent to the social optimum

- Assumptions
  - No entry: There are two firms and no entry by other firms is possible
  - Homogeneity: The firms produce products
  - Single period: This market and the two firms only exist for one period
  - Demand: The market demand curve is a linear function of price
  - Costs: Each firm has a constant marginal cost

Betrand Model[2]

- Mechanism
  - Suppose that Firm 1 charges a price p_1, which is greater than its marginal cost. If Firm 1 makes any sales at all, it earns a positive profit
  - Because both firms produce identical products:
    - ★ all consumers buy from Firm 2 if p_2 is even slightly below p_1
    - ★ none buy from Firm 2 if p_2 is above p_1
    - ★ consumers are indifferent between the two firms when $p_2 = p_1$

Betrand Model[3]

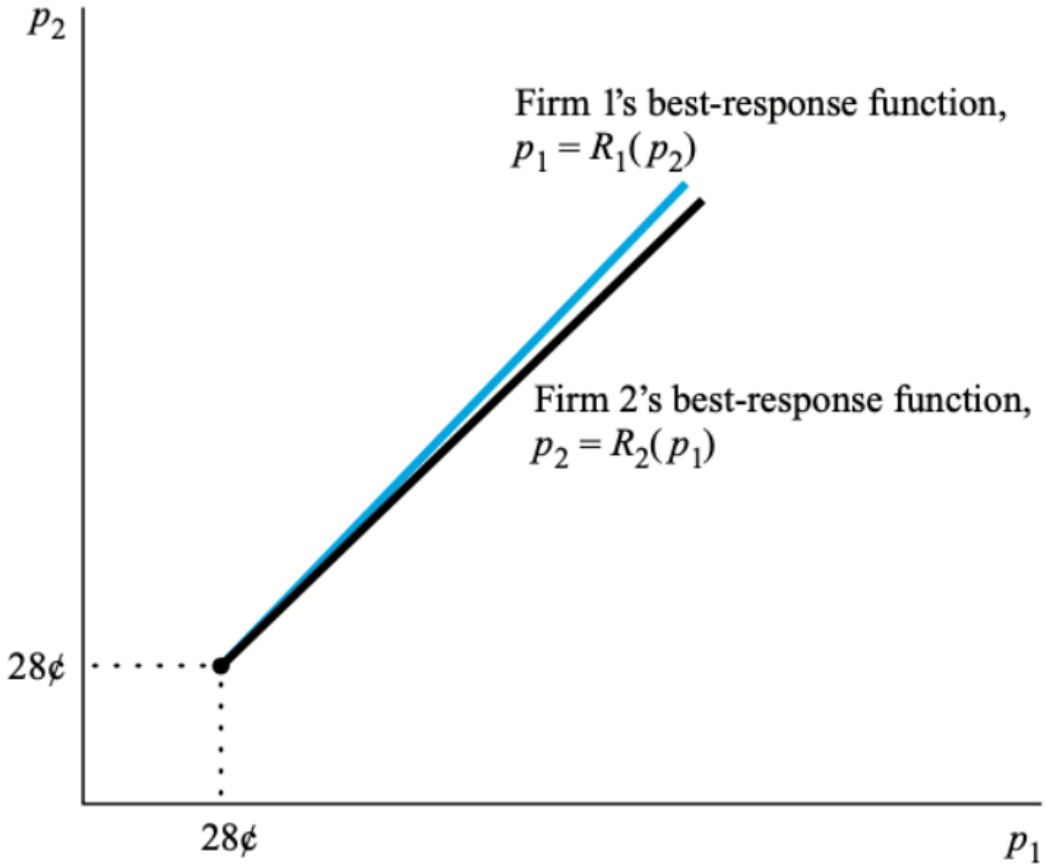


Betrand Model[4]

- Mechanism (cont'd)
  - The residual demand curve facing Firm 2
    - ★ is zero when $p_2 > p_1$
    - ★ equals the market demand when$p_2 < p_1$
    - ★ is horizontal when $p_2 = p_1$
  - If both firms charge the same price, they split the total market demand
    - ★ where the demand facing Firm 1 is horizontal (at $p_2 = p_1$) half the horizontal line is dashed to indicate that Firm 1 sells only half the total amount demanded
    - ★ neither firm profits by changing its price
    - ★ If a firm lowers its price, it loses money as price $<$ marginal and average cost
    - ★ If either firm raises its price, it makes no sales at all
    - ★ the only possible **Bertrand equilibrium** is $p = MC$

Betrand Model[5]

FIGURE 6.5 Bertrand Best-Response Functions



Betrand Model[6]

- Mechanism (cont'd)
  - Given whatever p_1 Firm 2 believes that Firm 1 will set, Firm 2 wants to set a p_2 slightly below p_1, as long as p_2 is greater than 28¢ (Marginal Cost)
    - ★ Firm 2's best-response function lies slightly below the 45° line (where the two prices are identical) through the point (28¢, 28¢)
    - ★ If Firm 1 sets p_1 below 28¢, Firm 2 does not respond because it cannot make a profit at any price
    - ★ Similarly, Firm 1's best-response function lies slightly above the 45° line and above 28¢
    - ★ The only intersection of these best-response functions is where price equals marginal cost
  - If both firms charge a price equal to marginal cost, they earn zero profits
    - ★ Bertrand equilibrium for homogenous goods is the same as the social optimum (competitive equilibrium)
    - ★ Consumers prefer the Bertrand equilibrium to the Cournot or cartel equilibria

Betrand Model[7]

- A Comparison of the Bertrand and Cournot Equilibria
  - Cournot equilibrium: With a small number of firms, output and price lie between the competitive and monopolistic equilibria
  - Bertrand equilibrium : As there are at least two firms, the Bertrand price is the competitive price (marginal cost)

Stackelberg Leader-Follower Model[1]

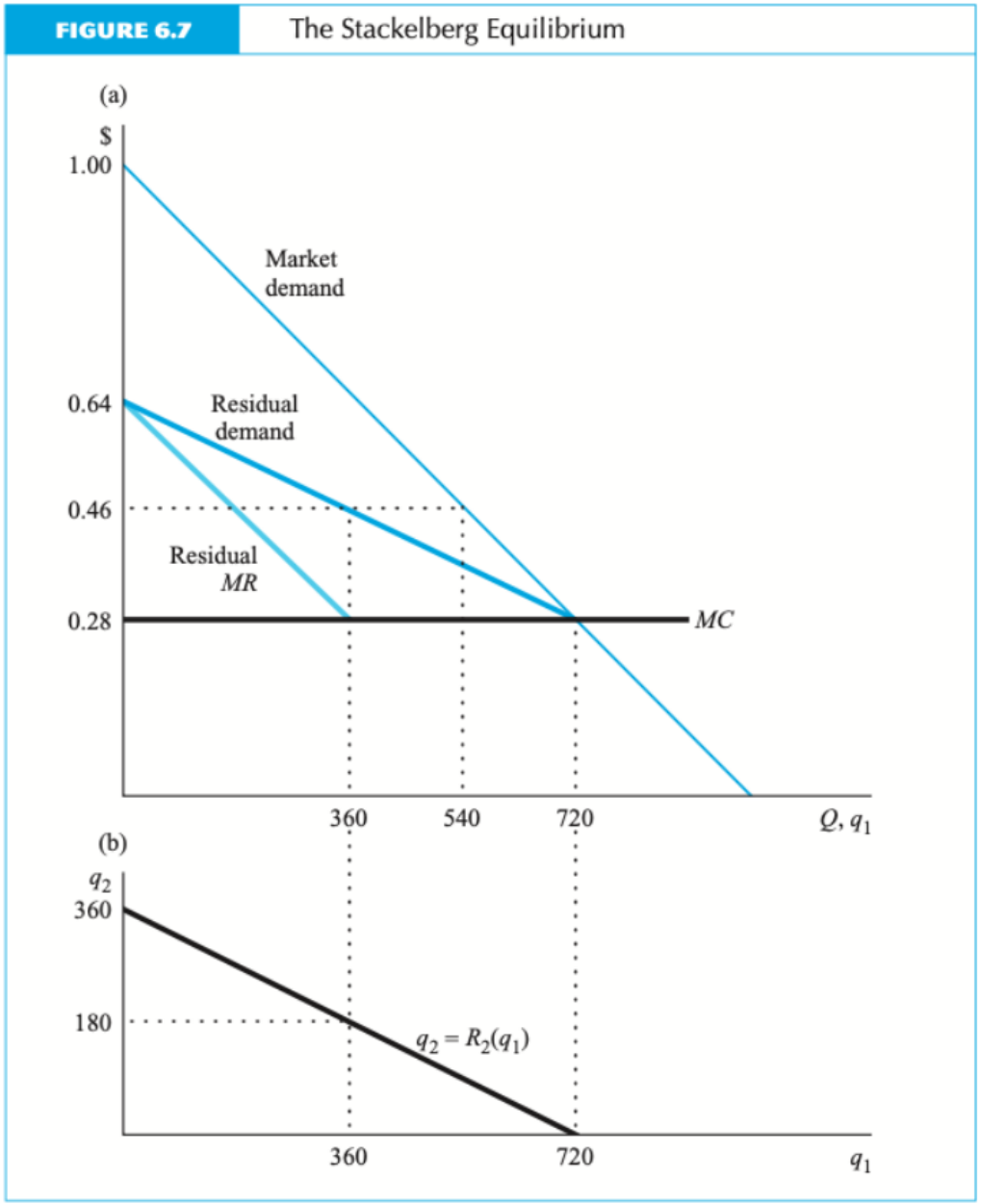
- Objective
  - In the Stackelberg model, firms set outputs, and one firm acts before the others
  - The leader firm picks its output level and then the other firms are free to choose their optimal quantities given their knowledge of the leader's output

- Assumptions
  - No entry: There are two firms and no entry by other firms is possible
  - Homogeneity: The firms produce products
  - Single period: This market and the two firms only exist for one period
  - Demand: The market demand curve is a linear function of price
  - Costs: Each firm has a constant marginal cost

Stackelberg Leader-Follower Model[2]

- Mechanism
  - Suppose Firm 2 is a follower and Firm 1 is the leader
  - Firm 1 realizes that once it sets its output (q_1), the firm 2 will use its Cournot best-response function to pick its optimal $q_2 = R_2(q_1)$
  - The leader, therefore, picks q_1 to maximize its profit subject to the constraint that firm 2 chooses its corresponding output using its Cournot best-response function

Stackelberg Leader-Follower Model[3]

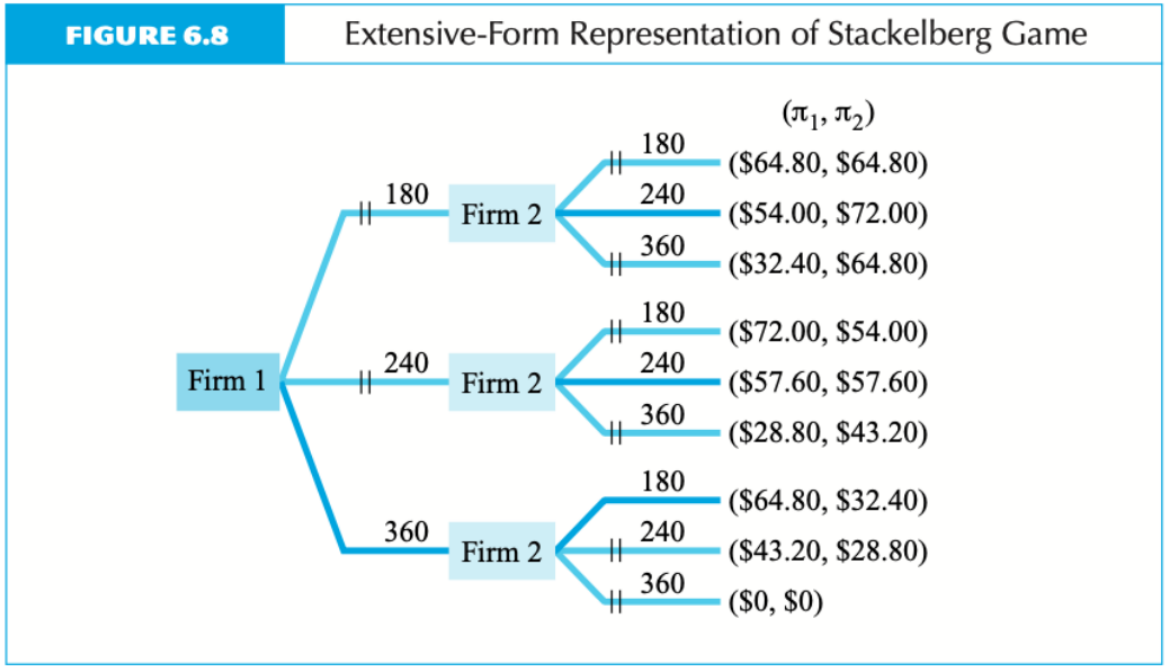


Stackelberg Leader-Follower Model[4]

- Mechanism (cont'd)
  - Because the firms have identical costs, Firm 1 knows the Cournot best-response function of Firm 2
    - ★ Consequently, the leader knows how much the follower will produce at any level of output the leader chooses
    - ★ Thus, the leader can calculate the total production corresponding to any output level it chooses, and it chooses the level that maximizes its profits
    - ★ By subtracting the follower's output from total demand, the leader calculates its residual demand curve. The leader picks its output q_1 where its marginal revenue based on its residual demand curve equals its marginal cost
    - ★ Firm 1 maximizes its profits by producing $q_1 = 360$ (Figure 6.7a)
    - ★ Firm 2 produces only $q_2 = 180$, which is determined by substituting 360 into Firm 2's best-response function (Figure 6.7b)

Stackelberg Leader-Follower Model[5]

- Mechanism (cont'd)
  - The Stackelberg game can be analyzed using the **extensive-form representation** of the game (or **decision tree**):



  - The extensive-form representation of the game shows the order in which firms make their moves, each firm's strategy at the time of its move, and the payoffs. There are an infinite number of combinations of outputs the two firms can produce

Stackelberg Leader-Follower Model[6]

- Stackelberg Equilibrium Compared to Other Equilibria
  - In the Stackelberg equilibrium, the leader is better off and the follower is worse off than in a Cournot equilibrium
    - ⋆ Stackelberg leader produces more output would a Cournot firm
    - ⋆ Stackelberg follower produces less output than would a Cournot firm
  - Total Stackelberg output is greater than the Cournot output , but less than the competitive equilibrium output
  - The Stackelberg price is higher than the competitive price, but lower than the Cournot price
  - Consumer surplus is higher with a Stackelberg duopoly than with a Cournot duopoly, but lower than the social optimum

A Comparison of the Major Oligopoly Models

- If there is only one firm, all three models predict monopoly behavior
- The more firms in the industry, the closer the Cournot and Stackelberg equilibria to the social optimum
- However, the Bertrand equilibrium with homogeneous goods is unaffected by the number of firms in the industry
  - As long as the market has at least two firms with unlimited capacity, the Bertrand oligopoly equilibrium is the same as the social optimum

Outline

Single-Period Prisoners' Dilemma Game[1]

- Suppose in the Cournot example that firms were restricted to choose one of only two possible output levels simultaneously: the firms can only produce the cartel output level or the Cournot level of output
  - Their actions and their payoffs, which depend on the strategies both choose, are summarized as follows:

		Firm 1: 240	Firm 1: 180
Firm 2	240	\$57.60, \$57.60	\$54.00, \$72.00
	180	\$72.00, \$54.00	\$64.80, \$64.80

- Each firm must choose its action or strategy without knowing what the other firm will do, i.e. the firms are engaged in **a game of imperfect information**
- If one strategy dominates all other strategies regardless of the actions chosen by rival firms, the firm should choose this **dominant strategy**

Single-Period Prisoners' Dilemma Game[2]

- What strategy should Firm 1 choose? The manager of Firm 1 could use the following reasoning:
  - If Firm 2 chooses the high-output strategy (240):
    - ⋆ If I choose high-output strategy also, my profit is \$57.60
    - ⋆ If I use my low-output strategy (180), I only earn \$54
    - ⋆ I prefer \$57.60 to \$54, so I'm better off with my high-output strategy
  - If Firm 2 chooses the low-output strategy (180)
    - ⋆ If I use my high-output strategy (240), my profit is \$72
    - ⋆ if I use my low-output strategy (180), my profit is only \$64.80
    - ⋆ Again, I'm better off with my high-output strategy
  - Therefore, whichever strategy Firm 2 uses, I'm better off using my high-output strategy: the high-output strategy is a dominant strategy
- The payoff table is symmetric, so the high-output strategy is also dominant for Firm 2

Single-Period Prisoners' Dilemma Game[3]

- Both firms use the high-output strategy, and this strategy is a **Nash equilibrium**: given the strategy of Firm 2, Firm 1 has no incentive to change its strategy and vice versa
  - This Nash equilibrium does not maximize the players' collective payoff
  - The two firms would be better off if they could cooperate and both use the high-price strategy
- This game is called a **prisoners' dilemma**: both firms have dominant strategies that lead to a payoff that is inferior to what they could achieve if they cooperated

Infinitely Repeated Prisoners' Dilemma Game[1]

- If this game is repeated, each firm can influence its rival's behavior by signaling and threatening to punish
  - use a multiperiod strategy of setting a low quantity (or high price) and taking losses for several periods to signal its willingness to collude
  - threaten to punish its rival if it does not collude

		Firm 1: 240	Firm 1: 180
Firm 2: 240		\$57.60 / \$57.60	\$54.00 / \$72.00
Firm 2: 180		\$72.00 / \$54.00	\$64.80 / \$64.80

Infinitely Repeated Prisoners' Dilemma Game[2]

- Example: how penalties can be used to insure collusion
  - Each of the two firms in the industry can produce at different output levels in different periods
    - ★ A possible strategy for a firm is to produce the Cournot-Nash level of output $q_n = 240$ each period
    - ★ If the other firm does the same, each earns the Cournot-Nash profits $\pi_n = \$57.60$ each period
    - ★ Alternatively, the firms can restrict output with each firm producing $q_m = 180$,and earning a profit of $\pi_m > \pi_n$ each period ($\pi_m = \$64.80$)
  - Firm 1 considers using the following two-part strategy:
    - ★ Firm 1 produces q_m output each period so long as Firm 2 does the same
    - ★ If Firm 2 produces a different level of output in any period t then in period $t + 1$ and thereafter, Firm 1 produces q_n

Infinitely Repeated Prisoners' Dilemma Game[3]

- Example: how penalties can be used to insure collusion (cont'd)
  - If Firm 2 believes that Firm 1 will follow this strategy, Firm 2 should produce q_m
    - Firm 2 knows that it can make greater profits in period t by producing more than q_m in that period
    - If Firm 2 does so, in the $t+1$ period and every period thereafter, Firm 1 would produce q_n
    - As already demonstrated, when Firm 1 produces q_n, Firm 2 maximizes its profits by producing q_n
    - Although Firm 2 can earn unusually high profits in period t,it earns relatively low profits for the rest of the time
    - Unless Firm 2 puts very little value on future profits, it is in Firm 2's best interest to tacitly collude and produce q_m in each period

- Since strategies can involve signals and threats of punishment, firms are more likely to charge the monopoly price in multiperiod than in single-period games

Types of Equilibria in Multiperiod Games

- The type of equilibrium in a repeated game depends on a player's ability to effectively threaten other players who are not cooperative. The effectiveness of a threat depends on
  - interest rate
  - the length of the game
  - credibility of the threat

Types of Equilibria in Multiperiod Games: Effectiveness of a Threat[1]

- If interest rates are so high that profits in future periods are worth substantially less than profits in the current period, future punishment is inconsequential and hence has no effect on current behavior
  - Lower interest rates, there fore, make the threat of punishment more effective
  - The more periods left in the game, the larger the total punishment as the punishment can be inflicted for more period
- However, if the threat is not credible, Firm 2 ignores the threat altogether
- Example: a two-peroid personer's dilemma example where two firms can choose any output level
  - Suppose Firm 1 is a cartel member and announces that it will
    - produce the collusive quantity q_m in Period 1
    - produce the Cournot-Nash quantity q_n in Period 2 if Firm 2 will produce the collusive quantity in Period 1
    - punish Firm 2 by producing a very large quantity ($q > q_n$) in Period 2 if Firm 2 produces more than q_m in Period 1

Types of Equilibria in Multiperiod Games: Effectiveness of a Threat[2]

- Example: a two-peroid personer's dilemma example where two firms can choose any output level (cont'd)
  - If Firm 2 believes that Firm 1 will carry out its threat, and the potential losses in Period 2 are large enough:
    - Firm 2 produces q_m in Period 1
  - However, Firm 2 does not view Firm 1's threat as credible and does not produce q_m in Period 1. Here is the reason:
    - There is now only one period left in the game: Firm 1 can punish Firm 2 in Period 2 by producing more than q_n, thereby lowering Firm 2's profits below the Cournot-Nash level
    - But will Firm 1 do that? Probably not
    - It is not in Firm 1's best interests to do so in Period 2: Firm 1 can only harm Firm 2 by harming itself and lowering each firm's profits below π_n
    - In Period 2, however, Firm 1 does not benefit from doing so. It is too late to affect Firm 2's behavior, and there are no future periods
    - Indeed, in Period 2, Firm 1 should act as though it is participating in a one-period game and produce q_n

Types of Equilibria in Multiperiod Games: Effectiveness of a Threat[3]

- Example: a two-peroid personer's dilemma example where two firms can choose any output level (cont'd)
  - Restrictions on possible equilibria are called **refinements**
    - One widely used refinement is to consider only perfect Nash equilibria: those Nash equilibria in which strategies (threats) are credible
    - For example, in the two-period game in which Firm 1 threatens to punish Firm 2 in Period 2 if Firm 2 produces too much in Period 1, the threat is credible only if the punishment is in Firm 1's best interest in Period 2
  - More generally, a strategy or threat is credible only if the firm will stick to that strategy in any subgame: a new game that starts in any period t and lasts to the end of the game
    - **subgame perfect Nash equilibrium**: the proposed strategies are best responses in any sub-game
  - If the period in which the game will end is not known until that period is over, a player is less likely to deviate from the cartel output level in that period
    - A game with a finite but unknown number of periods, so that players do not know which period is the final one, is therefore similar to a game with an infinite number of periods, and hence an enforceable cartel agreement is feasible

Outline

Setup

- Assume that there are n firms, where n is exogenously determined
- The output of the i th firm is q_i and the total output Q is the sum of the (homogeneous) output of each firm: $Q = q_1 + q_2 + ... + q_n$
- The demand and cost functions are as follows (a, b, and m are constants, demand is linear and marginal cost is constant):

	General Functional Form	Linear Example
Market demand	$p(Q)$	$p = a - bQ$
Firm's cost	$C(q_i)$	$C(q_i) = mq_i$

- The competitive and monopoly solutions are as follows (a, b, and m are constants):

	General Functional Form	Linear Example
Competition	$MC \equiv C'(q_i) = p(Q)$	$m = a - bQ = p$
		$Q = \frac{a-m}{b}$
Monopoly	$MC = C'(Q) = p'(Q)Q + p(Q) = MR$	$m = a - 2bQ = MR$
		$Q = \frac{a-m}{2b}$
		$p = \frac{a+m}{2}$

Analyze a Cournot Model[1]

- Firm 1 tries to maximize its profits through its choice of q_1

$$\max_{q_1} \pi_1(q_1, q_2, \cdots, q_n) = q_1 p(q_1 + \cdots + q_n) - C(q_1)$$

- The first-order condition is $MR = MC$ or

$$p(q_1 + ... + q_n) + q_1 p'(q_1 + ... + q_n)(1 + \frac{\partial q_2}{\partial q_1} + ... + \frac{\partial q_n}{\partial q_1}) = C'(q_1)$$

- If the firms play Cournot, these partial derivatives are zero. Thus, the first-order condition may be rewritten as

$$p(q_1 + ... + q_n) + q_1 p'(q_1 + ... + q_n) = C'(q_1)$$

  - Rearranging terms, multiplying and dividing the right-hand side by n. Lerner Index is as follows,

$$\frac{p - C'}{p} = -\frac{1}{n}\frac{dp}{dQ}\frac{Q}{P} = -\frac{1}{n\epsilon}$$

  - Lerner Index measures market power: the ratio of the price markup over marginal cost to the price
  - If the market is competitive, then $p = C'$, and Lerner Index is zero

Analyze a Cournot Model[2]

- Firm 1's best-response function:

$$q_1 = R_1(q_2, ..., q_n)$$

- With the linear example, the first-order condition for profit maximization is

$$MR = a - b(2q_1 + q_2 + ... + q_n) = m = MC$$

  - In equilibrium, all firms have the same cost function, $q_2 = q_3 = ... = q_n \equiv q$
  - The best-response function for the first firm is

$$q_1 = R_1(q_2, ..., q_n) = \frac{a-m}{2b} - \frac{n-1}{2}q$$

- The intersection of the best-response functions determines the Cournot equilibrium. Setting $q_1 = q$ and solving for q gives

$$q = \frac{a-m}{(n+1)b}$$

  - Total output: $nq = \frac{n(a-m)}{(n+1)b}$

Analyze a Cournot Model[3]

- The corresponding price is obtained by substituting $Q = nq$ into the demand function:

$$p = \frac{a + nm}{n + 1}$$

  - Setting $n = 1$ yields the monopoly quantity and price
  - As n becomes large, the quantity and price approach the competitive levels

Analyze a Stackelberg Leader-Follower Model[1]

- A Stackelberg leader (Firm 1) takes the Cournot best-response functions of the follower firms as constraints. That is, its objective is

$$\max_{q_1} \pi_1(q_1, q_2, ..., q_n) = q_1 p(q_1 + ... + q_n) - C(q_1)$$

$$s.t.\ q_i = R_i(q_i, Q_i) \qquad i = 2, ..., n$$

  - where Qi is the sum of the output of all the firms except Firm 1 and Firm i
- Substituting the best-response functions into the profit expression for each q_i and differentiating with respect to q_1, we obtain the first-order condition for a profit-maximum. For example, with a duopoly, the first-order condition for the Stackelberg leader is

$$p(q_1 + R_2(q_1)) + q_1 p'(q_1 + R_2(q_1))[1 + R_2'(q_1)] = C'(q_1)$$

  - where R_2' is the partial derivative of the best-response function of Firm 2 with respect to q_1
  - the follower's output is determined by setting the q_1 determined into the follower's best-response function

Analyze a Stackelberg Leader-Follower Model[2]

- Each follower's best-response function is

$$q_i = \frac{a-m}{2b} - \frac{(n-2)q}{2} - \frac{q_1}{2} \qquad i = 2, ..., n$$

  - where the output of the firms other than Firm i and Firm 1 is $(n-2)q$
  - because all the follower firms produce the same amount of output (q), the best-response function of the followers can also be written as

$$q = \frac{a-m}{nb} - \frac{q_1}{n}$$

- The leader maximizes its profits, taking the best-response functions of the followers as given. Substituting the linear demand curve expression for p into $p(q_1 + R_2(q_1)) + q_1 p'(q_1 + R_2(q_1))[1 + R_2'(q_1)] = C'(q_1)$ and solving for q_1, one obtains the output of the leader:

$$q_1 = \frac{a-m}{2b}$$

Analyze a Stackelberg Leader-Follower Model[3]

- In this linear model, q_1 is independent of the number of follower firms and equals the monopoly output. The output for the $n-1(\geq 1)$ follower firms is

$$q = \frac{a-m}{2bn}$$

- Thus, $q_1 > q_2$ for any number of firms $n \geq 2$. The total industry output is

$$Q = \frac{a-m}{2b}(\frac{2n-1}{n})$$

- Using the Cournot $q$from from $q = \frac{a-m}{(n+1)b}$. The market price is

$$p = \frac{a+m(2n-1)}{2n}$$

- Thus, as the number of firms (n) grows large, price and total quantity approach the competitive levels: $p \to m$ and $Q \to \frac{a-m}{b}$

Outline

1. Oligopoly
   - Assumptions
   - Terminology
   - Single-Period Oligopoly Models
   - Multiperiod Games
   - Mathematical Derivation - Cournot and Stackelberg Model

2. Product Differentiation and Monopolistic Competition
   - Differentiated Products
   - Location Model

Key Questions

- Key questions in this part:
  1. Why does product differentiation increase firms' market power?
  2. What number of firms maximizes welfare if all brands are perfect substitutes (homogeneous)?
  3. What number of firms maximizes welfare if consumers view brands as imperfect substitutes for each other?
  4. What number of firms maximizes welfare if consumers only value some of the brands in the market?

Outline

Key Concepts

- Two Key Concepts
  1. products are differentiated because consumers think they differ
     - ★ even though aspirin brands may be chemically identical, if consumers believe that the products differ and shop accordingly, then the products are effectively differentiated
  2. pricing of one brand exerts a greater constraint on another brand's pricing when the two brands are close substitutes than when they are not

The Effect of Differentiation on a Firm's Demand Curve[1]

- Oligopolies or monopolistic competition markets may have differentiated goods, but in a perfectly competitive market, products are not differentiated
  - If a firm's product is differentiated, it faces a downward-sloping demand function, which is inconsistent with a competitive firm's price-taking behavior.
    - ★ In industries with undifferentiated products: a firm's demand cruve depends only on the total supply of its rivals
    - ★ In an industry with differentiated products: a firm's demand cruve depends on the supply of each of its competitors separately
- For industries with either differentiated or undifferentiated goods, Firm i's inverse demand curve is

$$p_i = D(q_1, ..., q_n)$$

- Where products are differentiated, Firm i's demand curve can also be written as a function of the prices of each rival product

$$q_i = \tilde{D}(p_1, p_2, ..., p_n)$$

The Effect of Differentiation on a Firm's Demand Curve[2]

- With undifferentiated products, only total market output $Q = q_1 + q_2 + ... + q_n$ matters in determining the price p.In this case, the inverse demand equation may be written as

$$p_i = p = D(q_1 + q_2 + ... + q_n) = D(Q)$$

- Suppose there are two firms in an industry. If the two products are viewed by consumers as identical, the price each firm may charge ($p = p_1 = p_2$) might be written as

$$p = a - bQ = a - b(q_1 + q_2) = a - bq_1 - bq_2$$

  - where a and b are positive constants
- In contrast, if consumers view the products as imperfect substitutes, Firm 1's demand curve may be

$$p_1 = a - b_1 q_1 - b_2 q_2$$

  - where $a > 0$ and $|b_1| > |b_2|$
  - i.e. an increase in Firm 1's output has a greater effect on its price than an increase in Firm 2's output

Preferences for Characteristics of Products

- To illustrate how products can be compared by examining their characteristics, suppose the only important characteristic of a soft drink is how sweet it is. Soft drinks are located in "sweetness" space:

- **Characteristic space**: There is an axis showing the amount of each characteristic or attribute. Each brand can be located in this space according to its characteristics

The Representative Consumer Model[1]

In this representative consumer model, the typical consumer views all brands as equally good substitutes for each other; hence, brands are treated symmetrically. This representative consumer model can be used to examine industries with either differentiated or undifferentiated products

- A Representative Consumer Model with Undifferentiated Products
  - All brands have the same characteristics. This model differs from the oligopoly models only in the way the number of firms in the industry is determined

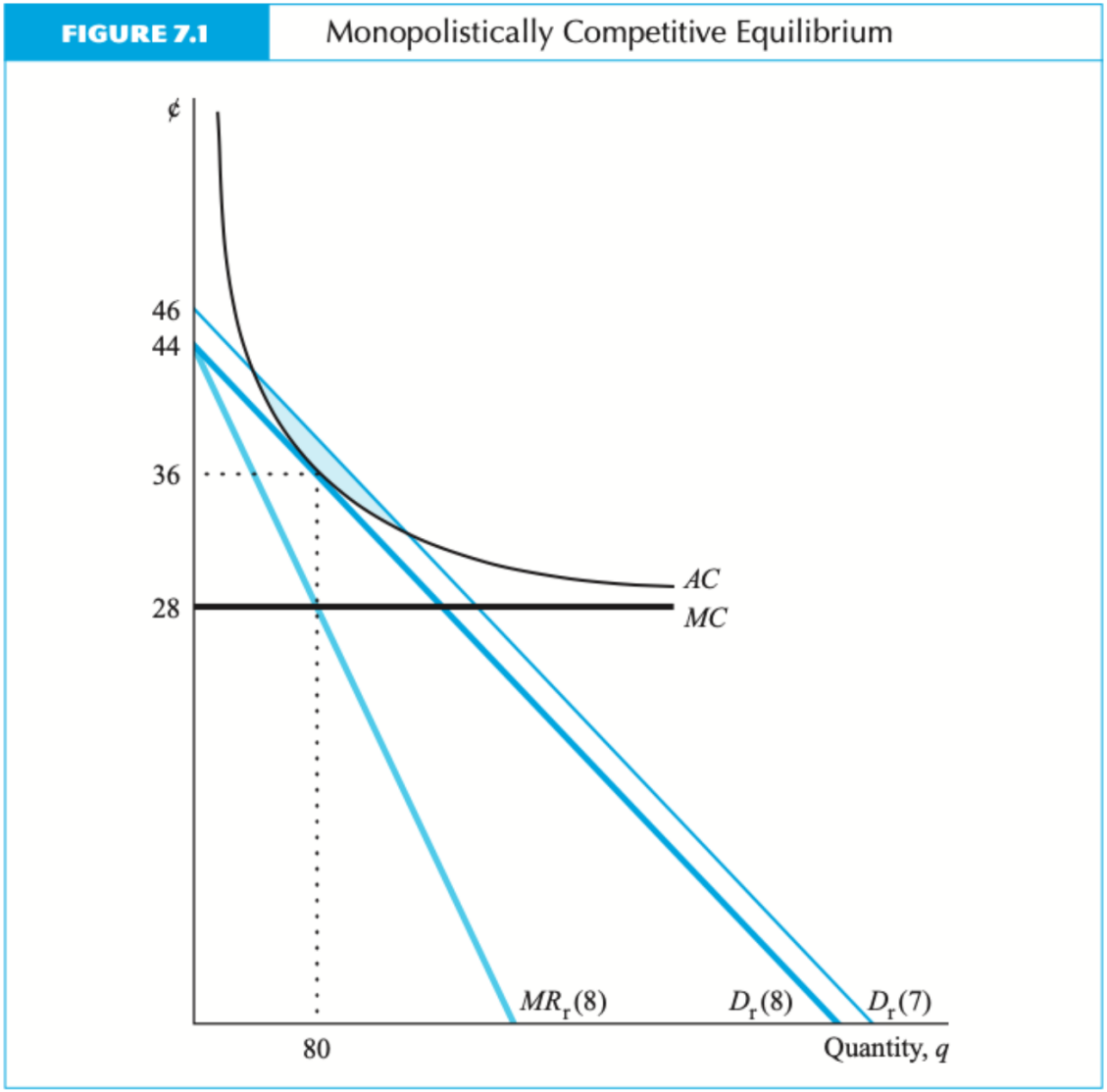
TABLE 7.1 **Comparison of Oligopoly and Monopolistic Competition Models**

Model	Profit Maximization by Individual Firms	Number of Firms (n) Determined by Entry
Noncooperative oligopoly	marginal revenue = marginal cost	No entry: number of firms is fixed at n
Monopolistic competition	marginal revenue = marginal cost	Free entry: firms enter until profit = 0, so n is endogenously determined

The Representative Consumer Model[2]

- A Representative Consumer Model with Undifferentiated Products (cont'd)
  - An example:
    - In equilibrium, no firm wants to change its output level, and each firm expects its rivals to produce at their actual level of output
    - Homogeneity: Output is homogeneous
    - Demand: The quantity that the market demands Q is a function of the market price p: $Q = 1000 - 1000p$
    - Costs: Each firm has a cost function of $C(q) = 0.28q + F$
    - Entry condition: Firms enter the market when profits are positive and exit when profits are negative

The Representative Consumer Model[3]



The Representative Consumer Model[4]

- A Representative Consumer Model with Undifferentiated Products (cont'd)
  - An example (cont'd):
    - ★ The average cost may be calculated by dividing total cost C (q) by output. That is,

$$AC = \frac{C(q)}{q} = 0.28 + \frac{F}{q}$$

    - ★ Firms enter the industry as long as profits are positive. Thus, firms enter the industry until economic profits are driven to zero:

$$\pi = pq - C(q) = 0$$

    - ★ Thus, in long-run equilibrium:
      - each firm makes zero profit overall
      - each firm's average cost equals its price $AC = p$

The Representative Consumer Model[5]

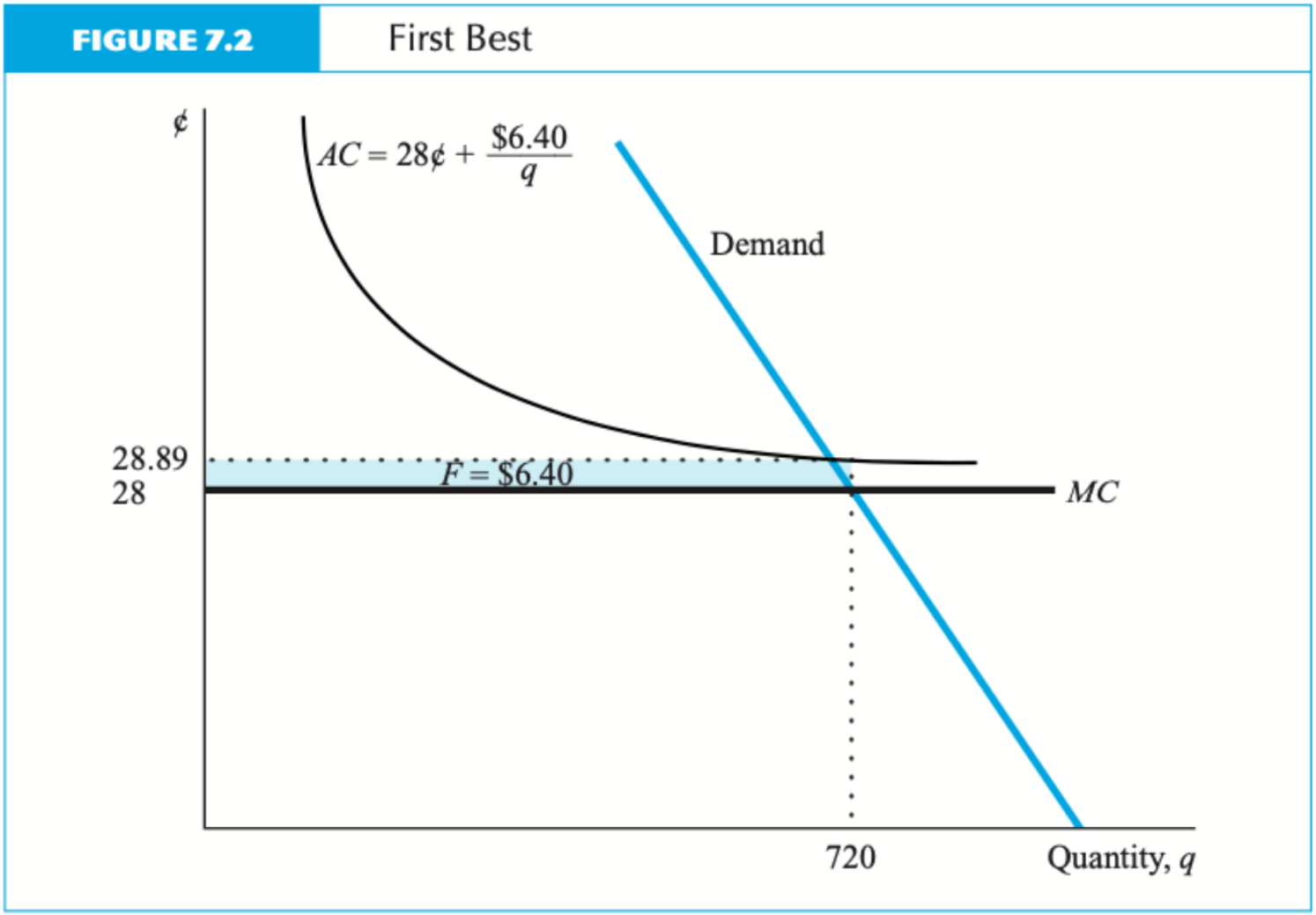
- A Representative Consumer Model with Undifferentiated Products (cont'd)
  - Lower Fixed Costs
    - ★ The lower the fixed costs, the higher the profits for any given number of firms in an industry. Additional firms must enter the industry to drive profits to zero
    - ★ Although fixed costs affect a firm's decision about whether to produce at all, they do not influence output levels if the firm actually produces:
      - each firm sets its output where $MR_r = MC$
      - neither MR_r nor MC are affected by a change in the firm's fixed cost
    - ★ $TC = VC + FC$
      - holding variable costs constant and lowering fixed costs causes total costs to fall
      - because total revenues remain constant as total costs fall, profits rise

The Representative Consumer Model[6]

- A Representative Consumer Model with Undifferentiated Products (cont'd)
  - Welfare with Undifferentiated Products
    - ★ Two welfare or efficiency problems arise with this monopolistic competition equilibrium:

1. as price is above marginal cost, the industry produces too little total output - an extra unit of this product is worth more to consumers than it costs firms to produce it
2. the number of firms is excessive when marginal costs are nonincreasing (constant or falling with quantity). Each additional firm must pay a fixed cost, F, so fixed costs to society are excessive

The Representative Consumer Model[7]



The Representative Consumer Model[8]

- A Representative Consumer Model with Undifferentiated Products (cont'd)
  - Welfare with Undifferentiated Products (cont'd)
    - ★ Society's optimal solution is to subsidize one firm to produce all the output and to require that price be set equal to marginal cost. The best possible solution (ignoring costs of administration) is referred to as the **first-best optimum**
    - ★ Typically, the government cannot regulate an industry so as to achieve a first-best solution and maximize society's welfare
      - e.g. it may be politically infeasible to subsidize a monopoly such as a local electric company
    - ★ By choosing the optimal number of firms, the government can achieve the **second-best optimum**: the best possible outcome subject to a constraint that violates one of the conditions for a first-best outcome, i.e. welfare is raised to the highest level possible given that the government does not subsidize firms
    - ★ By restricting entry, the government obtains the second-best optimum
      - although welfare is not as high as in the first-best optimum, it is higher than in the unrestricted, monopolistic competition equilibrium

The Representative Consumer Model[9]

- A Representative Consumer Model with Differentiated Products
  - Profit maximization is still determined by $MR_r = MC$
  - Entry still occurs only so long as profits are positive
  - The only modification to the model caused by product differentiation: firm's demand curve depends on the individual quantities produced by each of its competitors rather than on just the total quantity
    - ⋆ Although products are differentiated, the general form of the demand curves facing each firm is identical
      • due to product differentiation, a firm's price is more sensitive to changes in the quantity of its own product than to those of its competitors:

$$p_i = a - b_1 q_i - b_2 \sum_{j \neq i} q_j$$

      • where $\sum_{j \neq i} q_j$ means the sum of the output of all firms except Firm i
      • impact of differentiation is that each firm faces a more steeply downward-sloping demand curve
      • which gives the firm more market power—the power to raise price profitably above marginal cost

The Representative Consumer Model[10]

- A Representative Consumer Model with Differentiated Products (cont'd)
  - Welfare with Differentiated Products
    - ★ The optimal welfare solution changes when products are differentiated
    - ★ In general, a monopolistic competition equilibrium with differentiated products has two problems - neither the price nor the variety (number of brands) is optimal
    - ★ Two factors determine the variety in a monopolistic competition equilibrium:

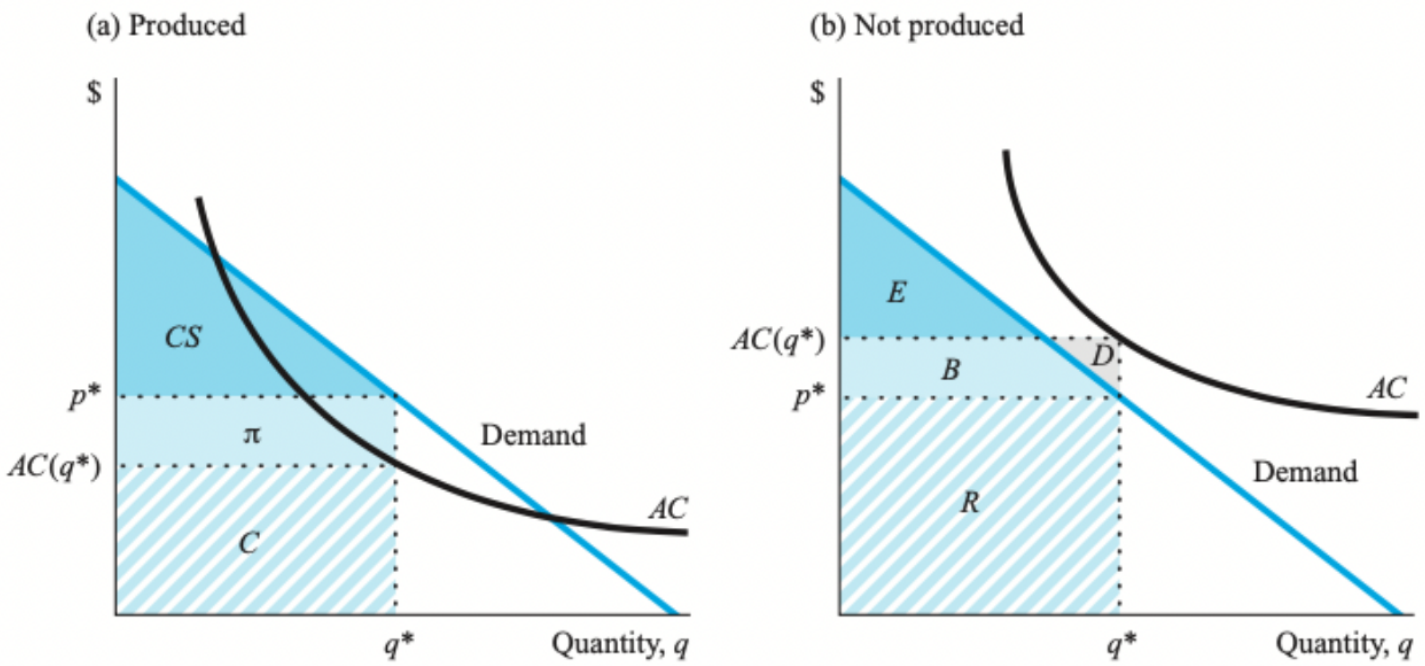
1. Highly desirable products may not be produced even though price is greater than firms' variable costs if fixed costs are so great that firms lose money
   - consumer surplus would rise if more products were produced, but the high fixed costs keep the number of brands below the optimal level
2. Effect on other firms is an offsetting force
   - when a firm introduces a new brand, it ignores the effect of its increased competition on the profits of other firms
   - when its product is a substitute for other brands, as Coke is for Pepsi, part of its profits come from these other brands
   - as firms ignore these effects on other firms, they have a tendency to produce too many products at too low prices

The Representative Consumer Model[11]

- A Representative Consumer Model with Differentiated Products (cont'd)
  - Fixed Costs Lead to Too Little Variety
    - ⋆ When firms operate in the increasing returns-to-scale section of their average cost curves, they tend to produce too few products
    - ⋆ If a firm's marginal cost does not rise rapidly, and it has large fixed costs, it operates in the downward-sloping, or increasing-returns, section of its average cost curve
      - Even though it is socially desirable to do so, the reason the product is not produced is that the firm does not obtain the entire social benefit even though it pays the entire social cos
  - The crucial point is that this distortion—the underproduction of certain products—is due to the presence of fixed costs and the firm's inability to capture consumer surplus

The Representative Consumer Model[12]

FIGURE 7.3 When Does a Market Produce a Product?



$CS + \pi + C$	Social Benefit $(CS + R)$	$E + B + R$
$CS + \pi$	Welfare $(CS + R - C)$	$E - D$
CS	Consumer Surplus	$B + E$
$\pi + C$	Revenue	R
C	Cost	$R + B + D$
π	Profit	$-(B + D)$

The Representative Consumer Model[13]

- A Representative Consumer Model with Differentiated Products (cont'd)
  - Optimal Diversity
    - ★ The optimal equilibrium reflects the trade-off between product variety, the number of brands, and the quantity of each brand produced, which is determined by the price
    - ★ For simplicity, assume that the number of brands n fully reflects the value of variety: more firms or brands, the better off are consumers, all else the same

TABLE 7.4 Variety and Quantity

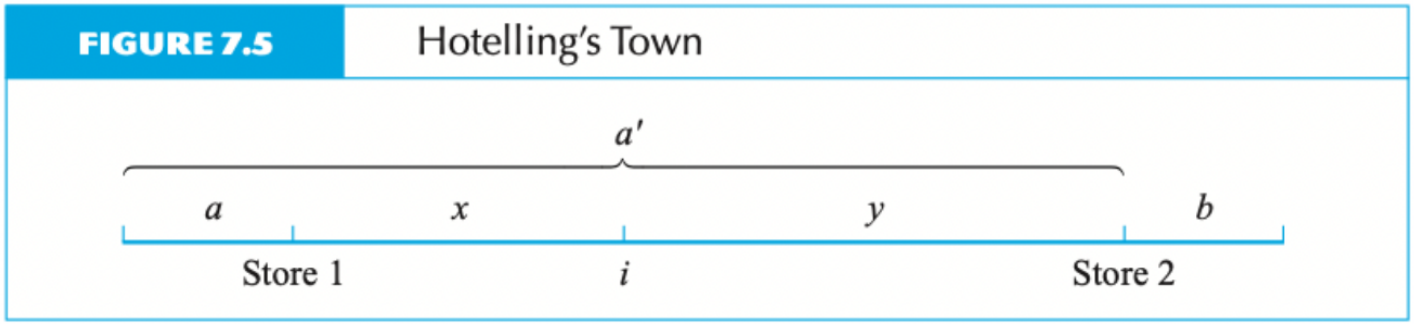
Number of Brands, n	Quantity of Each, q
1	95
2	45
3	28.33
4	20
5	15
6	11.67
7	9.29
8	7.5
9	6.11
10	5

Outline

Location Model

- Location models are monopolistic competition models in which consumers view each firm's product as having a particular location in geographic or product (characteristic) space
- The closer two products are to each other in geographic or characteristic space, the better substitutes they are

Hotelling's Location Model[1]



- Consider a long, narrow city with only one street, Main Street, that is a fixed length
  - Consumers are uniformly distributed along this street, so that in any block there are an equal number. All consumers are identical except for location, and each consumer buys 1 quart of milk in each time period
  - Suppose that the government sets the price of milk
  - How should Store 1 choose its location to maximize its profits if Store 2 is already located b miles from the right end of the city and cannot change its location?
    - ⋆ Store 1 maximizes its profits by locating just to the left of Store 2
  - If Firm 2 could costlessly relocate after Firm 1 locates, it would move slightly to the left of Firm 1's new location
    - ⋆ This process would be repeated until both firms were in the middle of the town, with each firm having half the customers.

Hotelling's Location Model[2]

- If price is given, the location of two firms can be determined:
  - This equilibrium is Nash in location strategies
- When firms can costlessly change their prices and their locations, there is a nonexistence of equilibrium

Salop's Circle Model[1]

Salop's Circle Model introduces two major changes in Hotelling's model:

1. Firms are located around a circle instead of along a line
   - A circle is roughly equivalent to an infinitely long line in that neither has end-points
   - It can be shown that a major cause of the nonexistence of equilibrium in Hotelling's model is the presence of end-points
2. Takes explicit account of a second or outside good
   - e.g. the differentiated product might be brands (flavors) of ice cream (the products located around the circle), and the outside good might be chocolate cake, which is an undifferentiated product competitively supplied by another industry

Salop's Circle Model[2]

- How Consumers Choose a Product
  - Assume that customers are uniformly located around the circle that is of unit circumference
  - For simplicity, each customer buys exactly one scoop of ice cream. A customer's location t^* represents that customer's most preferred type of ice cream
  - The utility a consumer gets from eating a scoop of a brand of ice cream located at t is

$$U(t, t^*) = u - c|t - t^*|$$

    - ⋆ where u is utility from the consumer's favorite flavor of ice cream; $|t - t^*|$ is the distance brand t is from the customer's favorite flavor t^*; and c is the rate at which a deviation from the optimal brand lowers the consumer's pleasure
  - Each consumer attempts to maximize consumer surplus, which is the difference between the consumer's pleasure from eating a brand located at t and the price: $U(t, t^*) - p$
  - **Best buy**: the product with the greatest surplus—the best combination of price and quality

Salop's Circle Model[3]

- How Consumers Choose a Product (cont'd)
  - If a chococlate cake (outside good) is a better buy in the sense that it gives more pleasure for a given amount of money. Suppose the surplus from the cake (pleasure from eating it less the price) is $\underline{u}$.The consumer only buys a scoop of the best-buy brand i of ice cream if its surplus is at least equal to $\underline{u}$

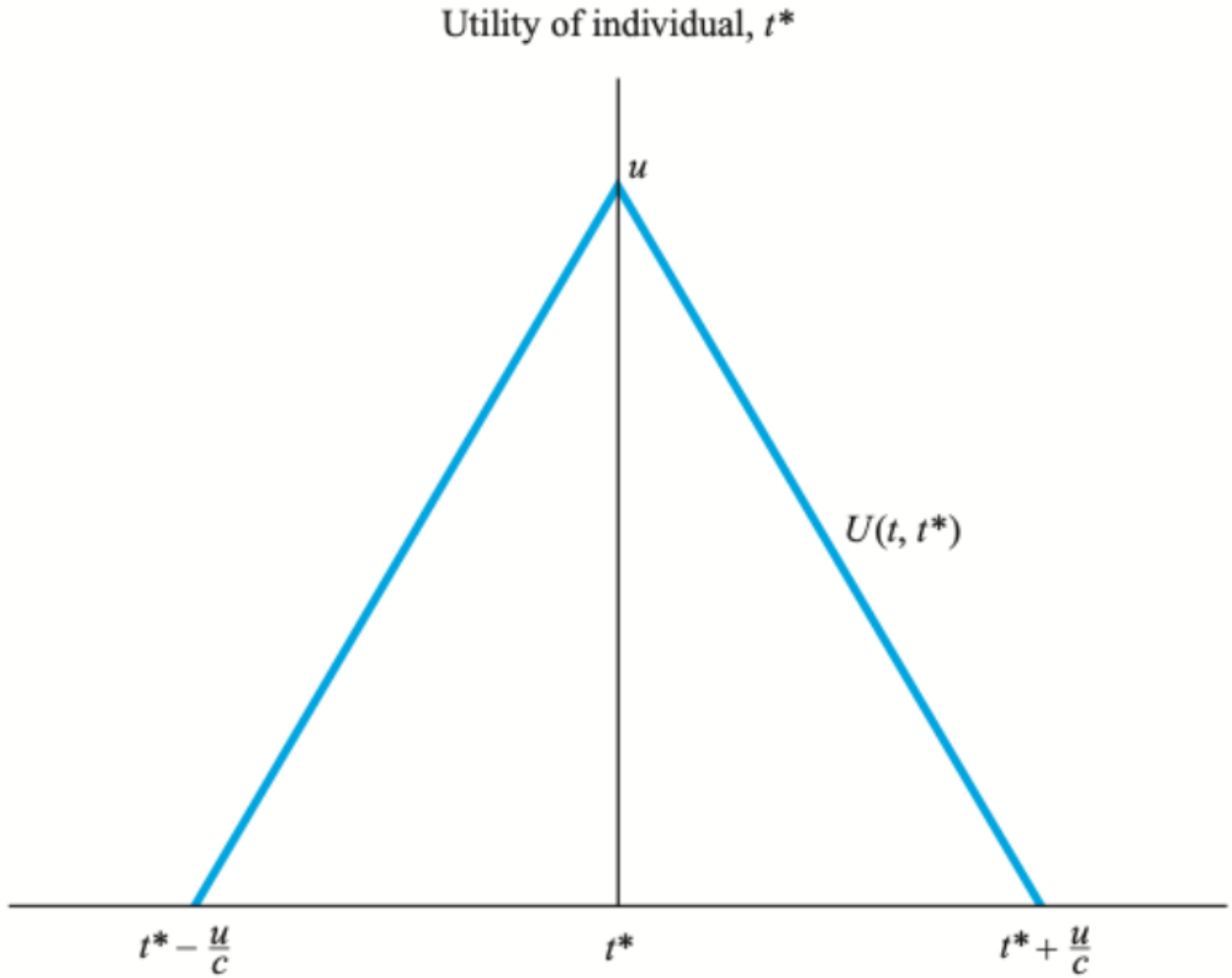
$$\max_i [U(t_i, t^*) - p_i] \geq \underline{u}$$

  - If a consumer's ideal ice cream is produced (located at t^*) and sold at p^*, the greatest surplus the consumer can get is $u - p^*$. The consumer is only willing to buy that brand if its surplus is equal to or greater than that from cake. That is, there is a reservation price $v = u - \underline{u}$ which is the highest price that the consumer is willing to pay for that brand of ice cream
    - ★ A consumer buys a scoop of ice cream only if the net surplus from the best-buy brand, the surplus from the best-buy brand minus the surplus from cake, is positive:

$$\max_i [v - c \mid t_i - t^* \mid - p_i] \geq 0$$

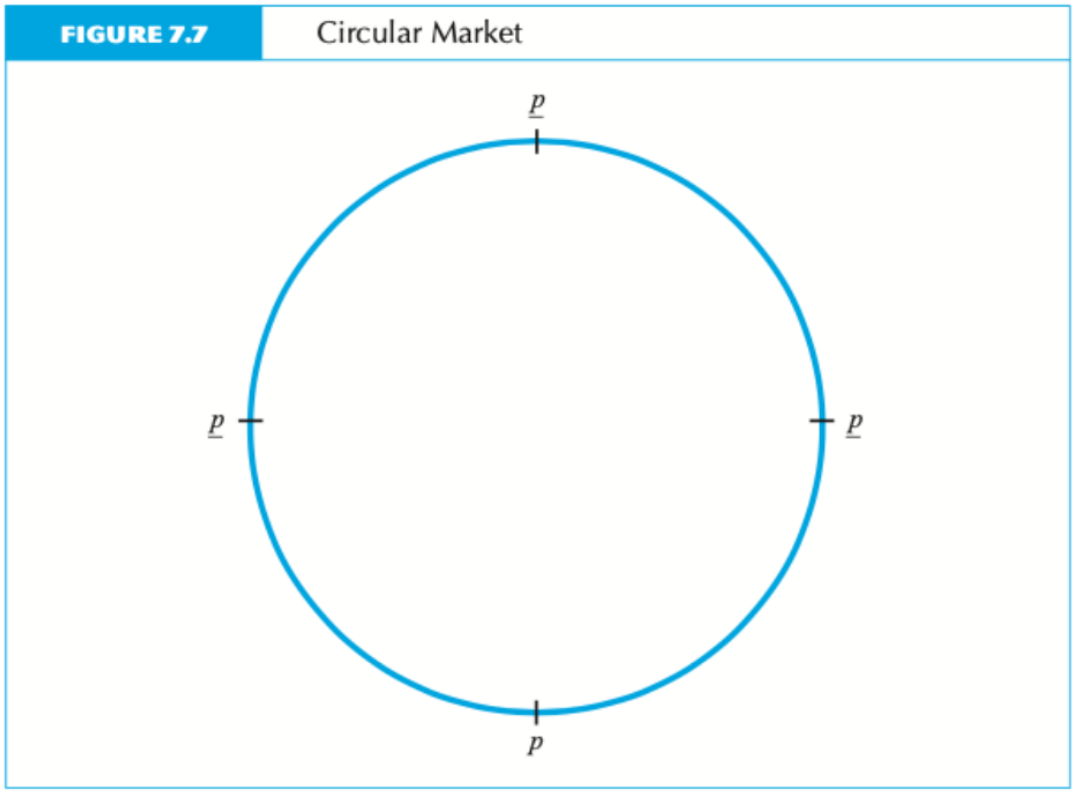
Salop's Circle Model[4]

FIGURE 7.6 Consumer's Utility Function



Salop's Circle Model[5]

- Firms' Behavior
  - Each firm wants to locate as far from its nearest competitors as possible
  - The further away other stores are from your store, the greater the market power you have with respect to the customers located near your store
    - ★ If there are n ice cream brands located at equal distances around the circle, the distance between two brands is 1/n (because the circle is of unit circumference)

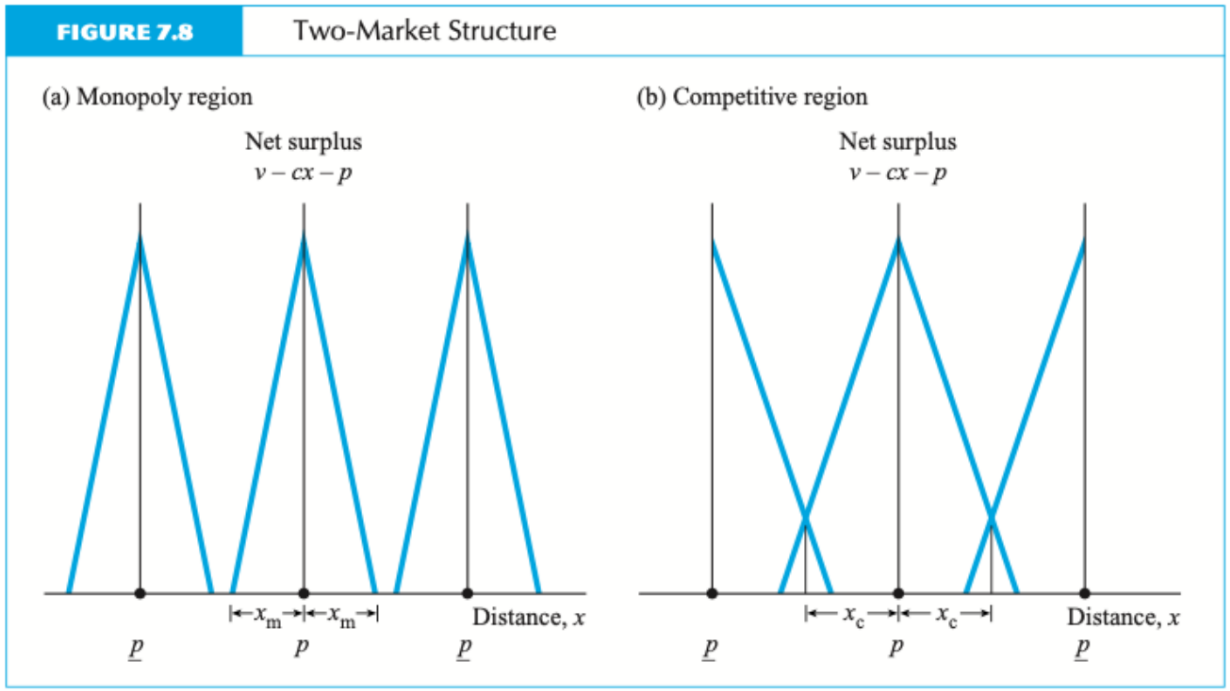


Salop's Circle Model[6]

- Monopoly Region
  - If there are relatively few brands, they do not compete with each other for the same consumers. Each brand is a local monopoly and sells to all consumers living close enough so that their net surplus is positive
  - Consider a consumer located a distance $x = |t - t^*|$ f from the brand at t with price p. The consumer is willing to buy that brand only if the consumer's net surplus is non-negative: $v - c_x - p \geq 0$
  - The maximum distance x_m a consumer can be located from that brand and still buy it is

$$x_m = \frac{v - p}{c}$$

Salop's Circle Model[7]



- Monopoly Region (cont'd)
  - The brand captures all the consumers who are no further than $x_m$distance on each side of its location, or all the consumers in a $2x_m$ segment of the circle
  - If there are L consumers located uniformly around the circle, the monopoly demand facing this brand, q_m, is $2x_mL$ or $q_m = \frac{2L}{c}(v - p)$

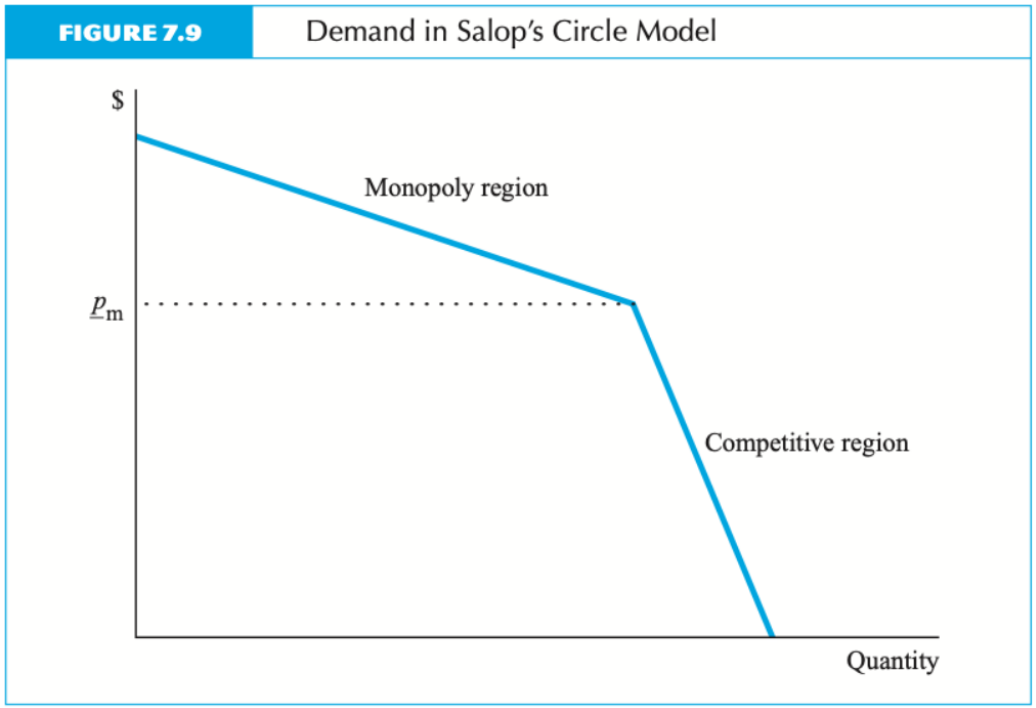
Salop's Circle Model[8]

- Competitive Region
  - If there are more firms,so that they are located closer together and compete for the same consumers, then each firm must take into account the price its rivals charge in setting its own price
  - Both of the typical brand's closest competitors are 1/n distance away and charge p. How much does this brand sell if it sets its price at p?

$$v - cx_c - p = v - c\left(\frac{1}{n} - x_c\right) - \underline{p}$$

  - Left side is the net utility from this brand, and right side is the net utility from the other brand

Salop's Circle Model[9]



- Competitive Region (cont'd)
  - Solving x_c and noting that the quantity demanded of a competitive firm is $q_c = 2x_c L$, the competitive demand equation is

$$q_c = \frac{L}{c}\left(\frac{c}{n} + \underline{p} - p\right)$$

    - ★ competitive quantity demanded falls by $-\frac{L}{c}$ as p rises by \$1 (holding $\underline{p}$ constant)

Salop's Circle Model[10]

- Salop shows that where firms have constant marginal and fixed costs, there exists a symmetric Nash equilibrium in which
  - no firm wants to alter its price
  - no additional firms want to enter.
- All firms charge the same price in equilibrium and are located 1/n distance from each other.
- Suppose that free entry is allowed and that firms can costlessly relocate so that they are equidistant from each other.
- Then, in a monopolistic competition equilibrium, the entry of one more firm causes all firms' profits to be negative.

Salop's Circle Model[11]

- Changes in Costs and Welfare in the Circle Model
  - In the circle model's monopolistic competition equilibrium, there are unambiguously too many brands; whereas in the differentiated products, representative consumer model equilibrium, there can be too many or too few brands.
  - Given the government's only regulatory policy is to control entry, the second-best optimum is either the market equilibrium or complete monopoly
    - That is, the optimal entry policy is either free entry or entry restricted so that each brand has a complete monopoly market

For Further Reading I

- Carlton, Dennis W., and Jeffrey M. Perloff. Modern Industrial Organization. Fourth edition. Harlow, Essex, England: Pearson, 2015. Print.
- Belleflamme, Paul., and Martin. Peitz. Industrial Organization : Markets and Strategies. Cambridge, UK ;: Cambridge University Press, 2010. Print.