Lecture 5: Price Discrimination[1]

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[1] The lecture is based on [Belleflamme and Peitz, 2015] Chapter 8(Group and personalized pricing) and Chapter 9.1(menu pricing under monopoly). We may introduce the concepts but not focus on the models and solutions from [Belleflamme and Peitz, 2015] Chapter 10 (Intertemporal price discriminatation) and Chapter 11(Bundling).

Motivation I

- So far, we assume a firm charges a uniform price for the whole market. With limited information and instruments, the best we can do is to "one-size-fits-all".
- Regarding the information about the consumers, it is not enough to know that consumers differ in their willingness to pay for our book; the crucial issue is to know who is willing to pay what.
- With such knowledge, we can increase profit by setting different prices to different consumers: **price discrimination**.
  - One important assumption is that reselling is not possible.

Motivation II

- Example of price Discrimination
  - personalized take-it-or-leave-it offer to each of them by quoting a price just below the consumer's reservation price.
  - segment the market on a geographical basis,
  - mechanism to let consumers reveal their willingness to pay, e.g. high-end version v.s. low-end version.
  - tariff instrument: The possibility of selling different versions of the same product is a first example of using more instruments than just prices.
  - A product that is sold over several periods makes it possible for a firm to set different prices over time.

Willingness to Pay

- Definition: WTP is the maximum price a consumer is willing to pay for a good.
- Knowledge of the WTP implies knowledge of demand curve
- Different consumers have different WTP.
- How do firms take advantage of this?

Price discrimination: a typology I

Following Pigou (1920), it is customary to distinguish three different types of price discrimination, according to the information that firms have about buyers.

- First-degree price discrimination (or perfect discrimination): The most favourable case for the firm is when it has complete information about individual preferences. The firm is then able to charge an individualized price for each buyer and for each unit purchased by each buyer, thereby extracting all of the consumer surplus.

Price discrimination: a typology II

- Third-degree price discrimination(Pigou's taxonomy), or as group pricing(Shapiro and Varian's): When the firm does not know exactly each consumer's willingness to pay, it may still manage to extract a fraction of the consumer surplus by relying on some indicators (such as age, occupation, location) that are related to the consumers' preferences.
- Second-degree price discrimination(Pigou's taxonomy), or as menu pricing: When buyers' characteristics are not directly observable, the firm still has the option to use self-selecting devices in order to extract some consumer surplus by targetting a specific package for each class of buyers.

"Know your customers"

- Personalized pricing has long been viewed as unworkable.
- Spatial economics was the only field in which pricing regimes akin to personalized pricing were explored: prices can be adjusted so as to include freight charges to the location of the buyer (so-called "delivered prices") and thus differ across buyers.
- ICT revolution has fuelled a rapid growth of computer-mediated transactions (e-commerce) and have increased the practicality of more personalized pricing regimes. Sellers can acquire precise information about their consumers through different sources
  - directly through repeated interaction with the buyers,
  - via telemarketing or direct-mail surveys,
  - from credit card reports, or
  - from web-based marketing firms (such as Double Click or I-Behaviour).

Case I

Data mining

The main idea behind data mining is to gather large amounts of customer information and to analyse it so as to identify trends indicative of customers' wants and needs. **Customer-loyalty** cards provide a great deal of useful information as they allow sellers to learn not only what is selling but also who is buying it. Data mining is now advancing on three fronts:

- (i) the use of data in real time in order to adjust prices,
- (ii) the use of historical data in order to predict future trends, and
- (iii) the analysis of "unstructured" data, such as text on the web.

Case II

As for the first direction (which is the one that directly relates to first- and third-degree price discrimination), a survey published in *The Economist* reports the following example. ”The traditional British pub seems like an unlikely place to find the latest in data mining. But some pub chains now change the prices of different drinks from day to day, using software that assesses the impact that ‘happy hour’ offers have on sales. If discounting a particular beer boosts sales one day, it is likely to remain discounted the next and if not, something else will be tried. As well as being much faster than traditional data mining, this kind of thing requires many other elements to be in place, such as the capacity to track inventory accurately and re-price products dynamically.”

Monopoly I

The model is specified as follow:

- A consumer's valuation (i.e., her reservation price) for the product, noted θ, is drawn from the uniform distribution over the interval $[0,1]$.
- At price p, consumers with $\theta \geq p$ purchase one unit of the product and the other consumers stay out of the market.
- Therefore, under our assumptions, the total quantity demanded is $q = 1 - p$.
- Suppose for simplicity that the producer's marginal cost is equal to zero.
- Then, if the firm is to set a uniform price, it chooses p so as to maximize $p(1-p)$.

Monopoly II

- We compute the firm's profits, the consumer surplus and the deadweight loss respectively as

$$\pi^u = \frac{1}{4}, CS^u = \frac{1}{8}, DL^u = \frac{1}{8}.$$

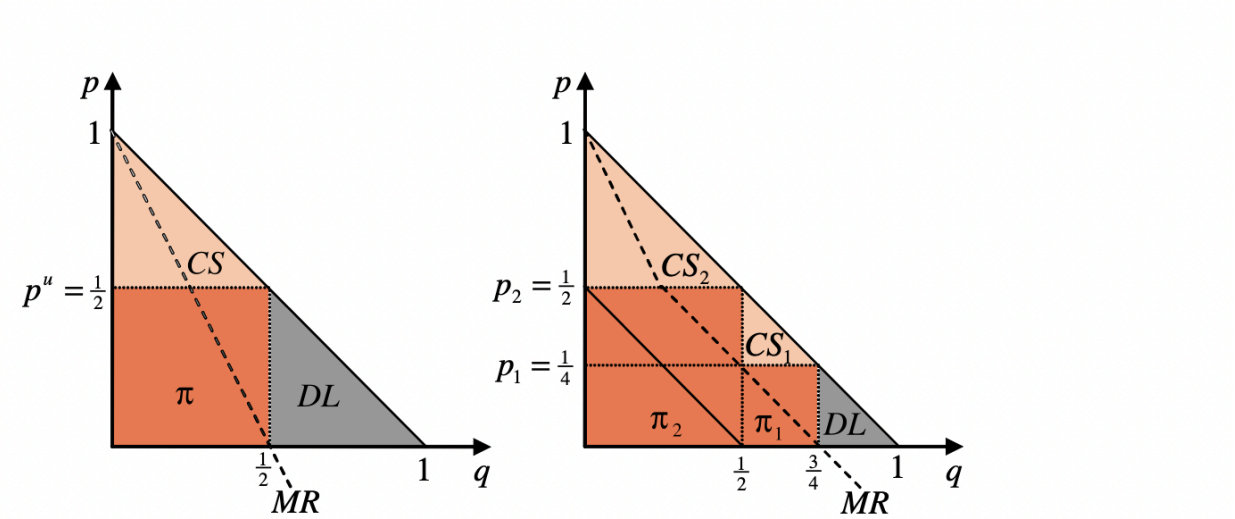


Figure 8.1 Uniform pricing (left) vs. group pricing with two segments (right)

Group Pricing I

- Suppose now that the monopolist acquires customer-specific information and, thereby, a more accurate estimate of how much each consumer values its product.
- In particular, the information partitions the $[0,1]$ interval into N subintervals of equal length and allows the monopolist to charge different prices to different groups of consumers. The larger N, the more precise the information;
- as N goes to infinity, prices can be personalized
- Monopolist is able to identify N segments with each segment having the length of $1/N$.

Group Pricing II

Consider the case $N = 2$, Now, facing a particular consumer, the monopolist knows whether this consumer's valuation lies between 0 and 1/2, or between 1/2 and 1.

- The firm sets a price p_1 for the first segment where demand is given by $q_1 = 1/2 - p_1$,
- and a price p_2 for the second segment where demand is given by $q_2 = max\{1/2, 1 - p_2\}$.
- The corresponding profits, consumer surplus and deadweight loss are (where we indicate in brackets the number of segments)

$$\pi(2) = \frac{1}{4} + \frac{1}{16} > \pi^u,$$

$$CS(2) = \frac{1}{8} + \frac{1}{32} > CS^u,$$

$$DL(2) = \frac{1}{32} < DL^u.$$

Group Pricing III

- $q_m = n/N - p_m$, $p_m = m/2N$.
- We conclude that an interior solution obtains on the first segment only. The profit, consumer surplus and deadweight loss are respectively given by:

$$\pi_1(N) = \left(\frac{1}{2N}\right)^2, CS_1(N) = DL_1(N) = \frac{1}{2}(\frac{1}{2N})^2.$$

Group Pricing IV

For all segments $2 \leq m \leq N$, the monopolist sets a price so as to cover the whole segmer $p_m = \frac{m-1}{N}$ and $q_m = \frac{1}{N}$. We have then

$$\pi_m(N) = \frac{1}{N}\left(\frac{m-1}{N}\right), CS_m(N) = \frac{1}{2}\left(\frac{1}{N}\right)^2, DL_m(N) = 0, m = 2, \ldots, N.$$

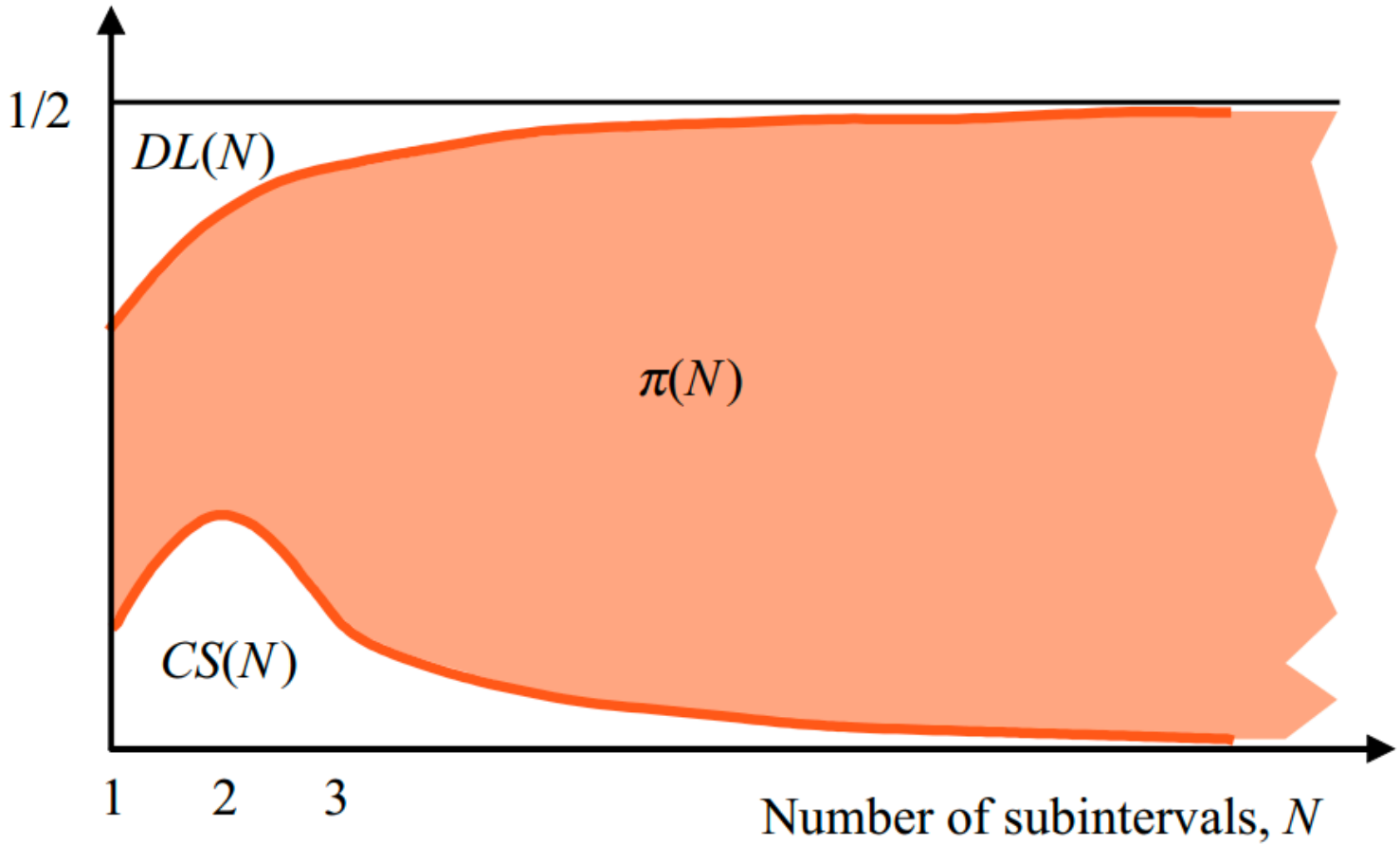
Summing over the N segments, we have

$$\pi(N) = \left(\frac{1}{2N}\right)^2 + \sum_{m=2}^{N} \frac{1}{N}\left(\frac{m-1}{N}\right) = \frac{2N^2-2N+1}{4N^2} = \frac{1}{2} - \frac{2N-1}{4N^2},$$

$$CS(N) = \frac{1}{2}\left(\frac{1}{2N}\right)^2 + (N-1)\frac{1}{2}\left(\frac{1}{N}\right)^2 = \frac{4N-3}{8N^2},$$

$$DL(N) = \frac{1}{8N^2}.$$

Group Pricing V



Monopoly Profit with Information

Group Pricing VI

As the information about the consumers' reservation prices becomes more precise, the discriminating monopolist increases its profits. The consumer surplus first increases but then decreases. In the limit case of personalized prices, the monopolist captures the entire surplus and the deadweight loss completely vanishes.

Group Pricing VII

Note that in terms of efficiency, personalized pricing (or first-degree, or perfect, price discrimination) by a monopolist is equivalent to perfect competition: it implements the first-best since the last unit is sold at marginal cost. However, in distributive terms, the two allocations are completely at odds with each other: the whole social surplus goes to the firm under personalized prices, but to the consumers under perfect competition.

Oligopolies I

The model

- Two firms, labelled 1 and 2, are located at the two endpoints of the unit interval;
- they sell competing brands (at zero marginal cost) to a unit mass of consumers who have unit demands and are uniformly distributed on $[0,1]$.
- Consumers have a reservation value r for their ideal product and incur a linear transportation cost, τ, per unit of distance. So, a consumer located at $x \in [0,1]$ derives utility $v_1 = r - \tau x - p_1$ when buying from firm 1 and utility $v_2 = r - \tau x - p_2$ when buying from firm 2.
- We assume that r is large enough, so that each consumer will buy.

Oligopolies II

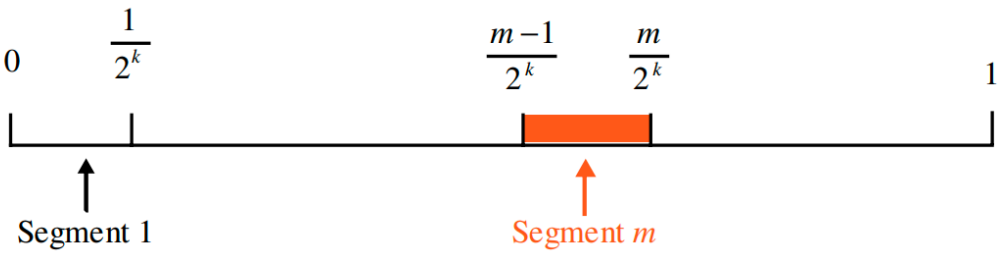


Figure 8.3 Partition of the unit interval

- In the standard Hotelling model, firms have no means to identify the location of any consumer.
- Here, we extend the analysis by letting firms acquire customer-specific information that allows them to classify the consumers into different segments and, thereby, to imperfectly estimate their location.
- As in the monopoly setting above, we assume that the information partitions the unit interval into N subintervals of equal length.

Oligopolies III

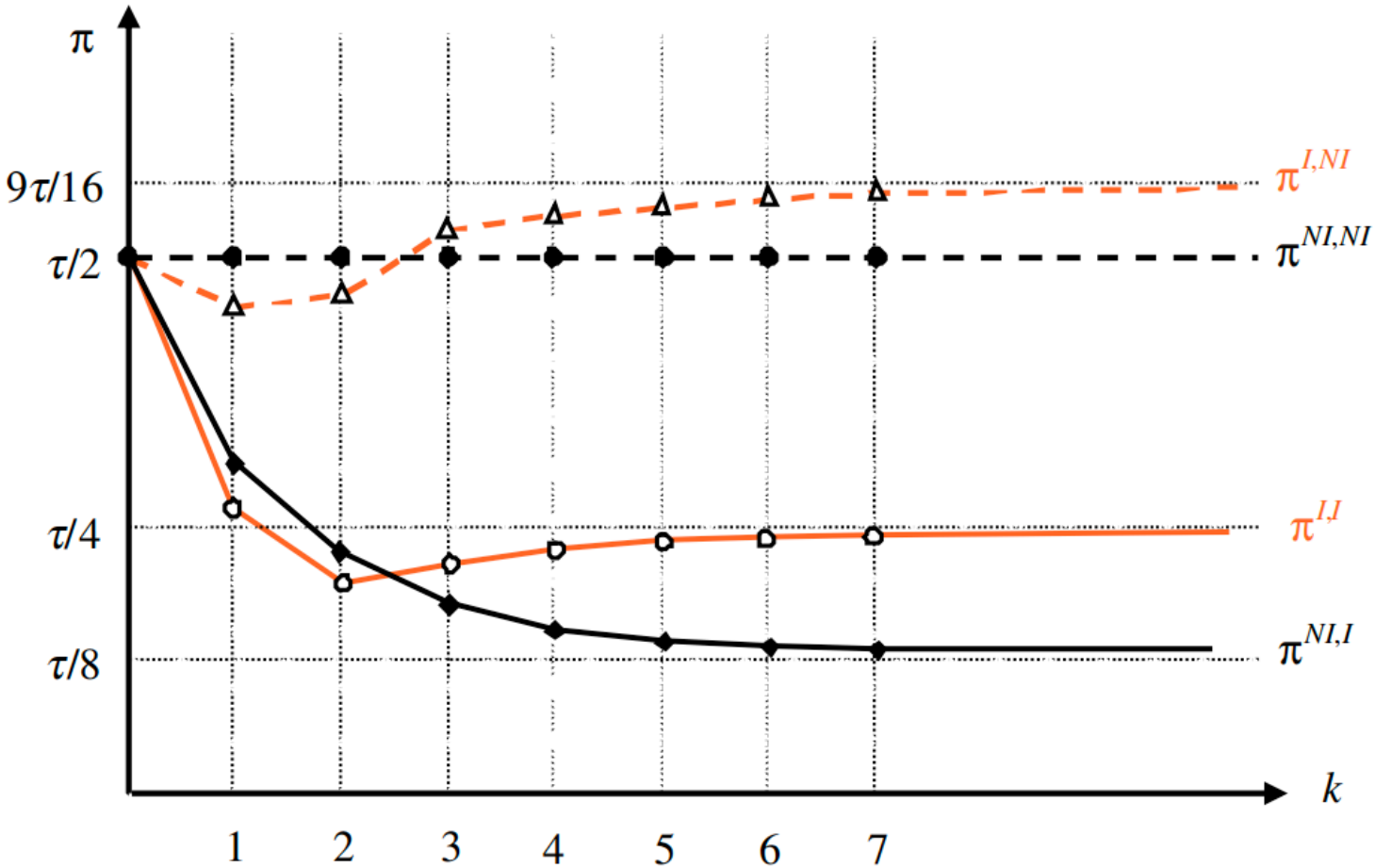
- We further assume that $N = 2^k, k = 0, 1, 2, ...$, where k can be seen as a measure of the quality of the information:
- the larger k, the finer the consumers' segmentation and the sharper the estimate of their valuation for the two brands.

Oligopolies IV

Pricing Decisions

- Neither firm acquires information.
  The "no information" profit $\pi^{NI,NI} = \frac{1}{2}\tau$.
- Both firms acquire information.
  The profit $\pi^{I,I}(k) < \pi^{NI,NI} = \frac{1}{2}\tau$.
- Only one firm acquires the information.

Oligopolies V



Oligopolies VI

Group pricing with imperfect competition

In a competitive setting, customer-specific information impacts firms in two conflicting ways.

- **Surplus extraction effect** it allows firms to extract more surplus from each consumer.
- **Competition effect** it exacerbates price competition.

When the quality of information is sufficiently large, the former effect dominates the latter. Then, firms use the information and price discriminate at equilibrium. However, they may well be better off if they could jointly agree not to use the information.

Monopoly

- The existence of price differences for the same products across countries or across regions is an issue that has attracted a lot of attention from economists.
- Macroeconomists wonder whether such differences invalidate the law of one price or the purchase parity power. Microeconomists wonder whether there is evidence of price discrimination.
- Answering the latter question requires separating the influence of cost differences and margin differences as the sources of price differences.
- Margin differences mainly result from differences in price elasticities across market segments.
- A monopoly which can prevent resale arbitrage will certainly price discriminate across segments with different price elasticities of demand.
- a monopoly will set the highest price in the segment

Monopoly

- A monopolist set its product on k separate markets.
- Let $Q_i(p_i)$ denote the distinct downward-sloping demand curve for market i.
- Let $C(q)$ denote the monopolist's total cost, with $q = \sum_{i=1}^{k} q_i$ and $q_i = Q_i(p_i)$.
- Monopolist maximize profit

$$\max_{(p_1,p_2,\cdots,p_k)} \sum_{i=1}^{k} p_i Q_i(p_i) - C\left(\sum_{i=1}^{k} Q_i(p_i)\right).$$

The monopoly pricing formula holds for all sections

$$\frac{p_i - C'(q)}{p_i} = \frac{1}{\eta_i},$$

where $\eta_i = \frac{-p_i Q_i'(p_i)}{Q_i(p_i)}$ is the elasticity of demand on market segment i.

Monopoly Geographic Price Discrimination

A monopolist optimally charges less in market segments with a higher elasticity of demand.

Oligopoly I

- Consider the following simple setting with three independent markets (indexed by 1, 2 and 3) and two chain-store retailers (indexed by 1 and 2).
- Retailer 1 is a monooplist on 1, retailer 2 is a monopolist on market 2, and the two retailers compete on market 3.
- assume that they are identical and that the inverse demand is given by $p_i = \alpha - q_i, i = 1, 2$.

Oligopoly II

Two stage game:

- In the first stage, the two retailers simultaneously choose between *local pricing* $p_i \neq p_{i3}$, and *uniform pricing*($\bar{p}_i = p_i = p_{i3}$) .
- At stage 2, firm i's maximization problem depends on its first-stage pricing choice.
  - Under *local pricing*,

$$\max_{p_i, p_{i3}} \pi_i^L = (\alpha - p_i)p_i + \frac{2}{3}(1 - 2p_{i3} + p_{j3})p_{i3}$$

Profit maximization prices: $p_i = \alpha/2$ and $p_{i3} = \frac{1}{4}(1 + p_{j3})$.

Oligopoly III

- Under *uniform pricing*,

$$\max_{\bar{p}_i} \pi_i^L = (\alpha - \bar{p}_i)\bar{p}_i + \frac{2}{3}(1 - 2\bar{p}_i + p_{j3})\bar{p}_i$$

Profit maximization price: $\bar{p}_i = \frac{2+3\alpha}{14} + \frac{1}{7}p_{j3}$

Oligopoly IV

Subgame		Local markets			Contested market		
		$\alpha = \frac{1}{2}$	$\alpha = \frac{2}{3}$	$\alpha = \frac{4}{5}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{2}{3}$	$\alpha = \frac{4}{5}$
LL		*0.25*	0.33	**0.40**	*0.33*	0.33	**0.33**
UU		*0.29*	0.33	**0.37**	*0.29*	0.33	**0.37**
LU, UL	L	*0.25*	0.33	**0.40**	*0.32*	0.33	**0.34**
	U	*0.30*	0.33	**0.36**	*0.30*	0.33	**0.36**

	Local pricing	Uniform pricing
Local pricing	π^{LL}, π^{LL}	π^{LU}, π^{UL}
Uniform pricing	π^{UL}, π^{LU}	π^{UU}, π^{UU}

Oligopoly

Geographic Price Discrimination with Oligopolistic Competition

Chain-store retailers are often in a monopoly position on small markets but compete with other retailers on larger markets. Despite the differences between the markets (and the absence of arbitrage), the retailers might refrain from price discriminating and set instead a uniform price on all markets. The reason is that uniform pricing acts as a *credible way to raise prices*, and thus to soften competition, on the contested markets.

Case I

Pricing by supermarkets in the United Kingdom
From April 1999 to July 2000, the UK Competition Commission carried out an extensive investigation of the supply of groceries from multiple grocery retailers (i.e., supermarket chains controlling 10 or more stores).

- Among the 15 leading supermarket groups, 8 were found to price uniformly while 7 adjusted prices to local conditions, such variation not being related to costs.

Case II

- For these 7 retailers, the Commission found that in general only a limited number of products had their price adjusted. As for the extent of the price differences, it was reported that ”the average level of difference between the minimum and maximum prices for each product varies across companies from 4.3 to 19.2 per cent, although the price range for individual products can be very much more, with prices as much as doubling in some stores compared with others“ (Competition Commission, p. 126).

Case III

- The Commission was concerned that local pricing may reflect the fact that shops were able to exploit local market power. Therefore, it considered remedying this potential problem by imposing national pricing or by requiring that prices be published on the Internet.
- However, simulation results led the Commission to conclude that such remedies were either undesirable, disproportionate or presented practical difficulties. Hence, the Commission made no recommendation for remedial action in respect of local pricing.

Case IV

- This seems much in line with the insights that we can draw from the model we developed above. It is clear that consumers of different markets are affected in opposite ways when going from uniform to local pricing. It is therefore not surprising that the impacts on global welfare are ambiguous

Motivation

- Group pricing pertains to situations where the sellers are able to infer their buyers' willingness to pay from some observable and verifiable characteristics of those buyers (age, gender, location, etc.).
- In many situations, however, there exists no such reliable indicator of the buyers' willingness to pay. How much a consumer is willing to pay is their private information.
- To extract more consumer surplus, the seller must offer his product under a number of "packages" (i.e., some combinations of price and product characteristics).
- The firms set pricing for different versions in such a way that consumers will sort themselves out by selecting the version that most appeals to them.

Case I

Examples of menu pricing in the information economy

- The dimension along which information goods are versioned is usually their quality, which is to be understood in a broad sense (for instance, the quality of a software might be measured by its convenience, its flexibility of use, the performance of the user interface, etc.).
- For instance, ”nagware” is a form of shareware that is distributed freely but displays a screen encouraging users to pay a registration fee, or displaying ads. In this case, annoyance is used as a discriminating device: some users will be willing to pay to turn off the annoying screen.

Case II

- Versioning of information goods can also be based on time, following the tactic of delay. For example, new books often appear first in hardcover and later as less expensive paperbacks.
- Similarly, movies can first be viewed in theaters; a few months later, they are released on DVD, and are shown on premium cable television; eventually, they are broadcast on terrestrial television. The price of these choices usually declines with the viewing date.

Case III

- Finally, versioning can be based on quantity: software site licences often provide discounted royalties as the number of networked machines or users grow; online databases offer discounts based on number of users or on usage by a particular user (measured by the number of searches performed, the quantity downloaded or printed, etc.); music performance licences use factors like the number of square metres in a bar or store, or the size of the audience for a radio or TV station to set quantity-based royalties; magazine and newspaper subscriptions feature quantity discounts.

Monopoly menu pricing I

$$v = \begin{cases} U(\theta, s) - p, & \text{if she buys one unit of } s \text{ at } p \\ 0, & \text{if she does not buy} \end{cases}$$

where $s \geq 0$ is the quality of the product, $p \geq 0$ is its price, and θ is the consumer's taste parameter. As all consumers value higher quality, we assume that $U(\theta, s)$ is an increasing function of s.

- For simplicity, we assume there are two types of consumers θ_1 (low type) and θ_2 (high type), in respective proportions $1 - \lambda$ and λ.

Monopoly menu pricing II

- We assume that high-type consumers care more about quality than low-type consumers: for any s, $U(\theta_2, s) > U(\theta_1, s)$.
- **Single crossing property**
  We assume that high-type consumers value more any increase in quality than low-type consumers: for any $s_2 > s_1$,

$$U(\theta_2, s_2) - U(\theta_2, s_1) > U(\theta_1, s_2) - U(\theta_1, s_1). \qquad \text{(SC)}$$

- The monopolist is able to produce two exogenously given qualities s_1 and s_2, at respective constant unit costs c_1 and c_2,$c_i < U(\theta_i, s_i)$.

Pooling Equilibrium I

The question is whether the monopolist will choose to offer the two qualities priced appropriately (menu pricing), or a single quality.

- Assume that the monopolist always prefers to offer the high quality s_2. To ensure this, the sufficient condition is

$$U(\theta_1, s_2) - U(\theta_1, s_1) > c_2 - c_1 \qquad \text{(HQ)}$$

- Then, the monopolist has two options:
  - charges the high price equal to $U(\theta_2, s_2)$ and sells to high-type consumers only,
  - lowers the price to $U(\theta_1, s_2)$ and sells to all consumers.

Pooling Equilibrium II

- The former option is more profitable if the proportion of high types, λ, is large enough, namely if $\lambda > [U(\theta_1, s_2) - c_2]/[U(\theta_2, s_2) - c_2] \equiv \lambda_0$.
- The profit from selling only the high-quality can thus be written as $\Pi_s = \begin{cases} \lambda(U(\theta_2, s_2) - c_2), & \text{if } \lambda \geq \lambda_0 \\ U(\theta_1, s_2) - c_2, & \text{if } \lambda < \lambda_0 \end{cases}$.

Separating Equilibrium I

Under menu pricing, the monopolist must find the profit-maximizing price pair (p_1, p_2) that induces type i consumers to select quality s_i. There are two concerns: participation (each consumer must do at least as well consuming the good as not consuming it) and self-selection (or incentive compatibility, each type of consumer must prefer their consumption to the consumption of the other type of consumer).

Separating Equilibrium II

- For the low-type group, the participation and incentive compatibility constraints respectively read as:

$$U(\theta_1, s_1) - p_1 \geq 0 \quad \text{(PC1)}$$

$$U(\theta_1, s_1) - p_1 \geq U(\theta_1, s_2) - p_2 \quad \text{(IC1)}$$

- Similarly for the high-type group:

$$U(\theta_2, s_2) - p_2 \geq 0 \quad \text{(PC2)}$$

$$U(\theta_2, s_2) - p_2 \geq U(\theta_2, s_1) - p_1 \quad \text{(IC2)}$$

Separating Equilibrium III

The monopolist wants to choose p_1 and p_2 to be as large as possible.
The equilibrium pricing is characterized by

$$p_1^* = U(\theta_1, s_1), \text{ and}$$
$$p_2^* = U(\theta_2, s_2) - [U(\theta_2, s_1) - U(\theta_1, s_1)]$$

Conclusion

Monopoly menu pricing

Consider a monopolist who offers two pairs of price and quality to two types of consumers. Prices are chosen so as to fully appropriate the low-type's consumer surplus. High-type consumers obtain a positive surplus (the so-called *"information rent"*) because they can always choose the low-quality offering instead.

When is menu price optimal? I

Consider the first case ($\lambda \geq \lambda_0$).
Compare profits when the monopolist only sells the high quality and when it price discriminates by selling both qualities.

- Profit with menu pricing: $\Pi_m = (1-\lambda)[U(\theta_1, s_1) - c_1] + \lambda[U(\theta_2, s_2) - (U(\theta_2, s_1) - U(\theta_1, s_1)) - c_2]$.
- Profit with a single quality: $\lambda(U(\theta_2, s_2) - c_2)$.

Then, menu pricing modifies profits as follows:
$\Delta\Pi = \Pi_m - \Pi_s = (1-\lambda)[U(\theta_1, s_1) - c_1] - \lambda(U(\theta_2, s_1) - U(\theta_1, s_1))$.

When is menu price optimal? II

Menu pricing involves two opposite effects.

- **market expansion** low-type consumers now buy the low quality, which yields a margin of $U(\theta_1, s_1) - c_1$ per consumer.
- **cannibalization**: high-type consumers still buy the high quality but now, at a price reduced by $U(\theta_2, s_1) - U(\theta_1, s_1)$.

When is menu price optimal? III

- The net effect is positive if λ is not to large: $\Delta\Pi > 0 \Leftrightarrow \lambda < [U(\theta_1, s_1) - c_1]/[U(\theta_2, s_1) - c_1] \equiv \bar{\lambda}_0$
- The latter condition is compatible with our starting point if and only if $\bar{\lambda}_0 > \lambda_0$, which is equivalent to $[U(\theta_2, s_2) - c_2]/[U(\theta_2, s_1) - c_1] > [U(\theta_1, s_2) - c_2]/[U(\theta_1, s_1) - c_1]$.

When is menu price optimal? IV

Consider the second case ($\lambda < \lambda_0$).

The change in profits induced by menu pricing is then given by

$$\Delta\Pi = \Pi_m - \Pi_s = (1-\lambda)[(U(\theta_1, s_1) - c_1) - (U(\theta_1, s_2) - c_2)] + \lambda[(U(\theta_2, s_2) - U(\theta_2, s_1)) - (U(\theta_1, s_2) - U(\theta_1, s_1))].$$

- There are again two opposite effects:
  - profit from low-type consumers decreases (because they buy the low quality instead of the high quality, which is detrimental for the monopolist according to assumption (HQ)),
  - profit from high-type consumers increases (they continue to buy the high quality but pay now a higher price according to assumption (SC)).

When is menu price optimal? V

- Here, the net effect is positive as long as high-type agents are numerous enough:
  $\Delta\Pi > 0 \Leftrightarrow \lambda > [U(\theta_1, s_2) - U(\theta_1, s_1) - (c_2 - c_1)]/[U(\theta_2, s_2) - U(\theta_2, s_1) - (c_2 - c_1)] \equiv \underline{\lambda}$.
- For this condition to be compatible with our starting point, we need $\lambda_0 > \underline{\lambda}$, or
  $U(\theta_2, s_2) - c_2]/[U(\theta_2, s_1) - c_1] > [U(\theta_1, s_2) - c_2]/[U(\theta_1, s_1) - c_1]$.

Note that this is the exact same condition as in the previous case: *going from low to high quality increases surplus proportionally more for high-type consumers than for low-type consumers.*

Conclusion

When is menu pricing optimal?

Menu pricing is optimal (i) if the proportion of high-type consumers is neither too small nor too large, and (ii) if going from low to high quality increases surplus proportionally more for high-type consumers than for low-type consumers.

References I

Belleflamme, P. and Peitz, M. (2015). *Industrial organization: markets and strategies*. Cambridge University Press.