

Lecture :
Differentiation

Jasmine Hao

Views of Product
Differentiation

Views on Product
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Horizontal
Product

Differentiation
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Differentiation

Lecture 3: Product Differentiation

Jasmine Hao

Hong Kong University

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What is product differentiation?

In most market, there exist no identical product from the view of consumers.

Questions about how to apprehend product differentiation:

- ▶ how do consumers perceive product and services?
- ▶ How similar are demand curves of various individuals?
- ▶ Do consumers care for variety?

To answer these questions

- ▶ Need modelling choices of consumers.
- ▶ Need to model firms' choice of product quality:
 - balancing between product that fits most tastes(**market size effect**)
 - avoid competition from the other firms(**competition effect**)

Example

- ▶ Car Market
 - each consumer buy one car
 - heterogeneous taste
- ▶ Beer/ Carbonated soft drink
 - unit demand and heterogeneous taste
 - differ in the brand preference
 - also different in individual curve

Views on Product Differentiation

Sources of product differentiation:

- ▶ heterogeneity among consumers - characteristics space
 - discrete choice approach
 - e.g. Hotelling model
- ▶ each consumer has a variable demand for all products, all consumers are identical
 - representative consumer approach
 - e.g. linear demand model

Example of differentiation I

A product can be differentiated based on:

- ▶ **Price:** The price is the most common determinant of which target group will be attracted to a brand's product. It separates a premium product from economical products. Example: Zara's products are considered premium products.
- ▶ **Features:** Features like size, shape, ingredients, origin, etc. differentiate products in the same price spectrum. They also help the brand to back their high pricing decisions.

Example of differentiation II

- ▶ **Performance & Quality:** A good quality product always stands out from standard quality products. Example: Duracell lasts 10 times longer than ordinary batteries.
- ▶ **Reliability:** Some products are known to be more reliable than others. That is, there is a less probability of them malfunctioning or failing within the given time period.

Example of differentiation III

- ▶ **Looks:** Looks play a very important role in differentiating the product especially in the case of apparels and other luxury products. Channels of Distribution: Channels of distribution also plays a vital role in differentiating a product from the competition. For example, Amway uses a selective distribution strategy to position itself as a quality brand.
- ▶ **Complexity:** The level of complexity of usage of a product plays a very important factor in differentiating products, especially in the technology industry.

Example of differentiation IV

- ▶ **Location:** Manufacturer's location, brand's home country, and retailers' location play an important role in differentiating a product from its competitors.
- ▶ **Marketing efforts:** Marketing efforts give rise to the brand image which is a decent product differentiator. Other marketing efforts like sales promotion act as an add-on to differentiation strategy. **After-sale services:** Good after sale services make the customers have faith in the brand and make them differentiate it from others.

What are Types of Product Differentiation? I

- ▶ **Horizontal Differentiation** Horizontal differentiation refers to any differentiation that is not associated with the product's quality or price point. Instead, these products offer the same thing at the same price point. When making decisions regarding horizontally differentiated products, it often boils down to the customer's personal preference.
- ▶ **Examples of Horizontal Differentiation:** Pepsi vs. Coca-Cola, bottled water brands, types of dish soap.

What are Types of Product Differentiation? II

- ▶ **Vertical Differentiation** In contrast to horizontal differentiation, vertically differentiated products are extremely dependent on price. With vertically differentiated products, the price points and marks of quality are different. And, there is a general understanding that if all the options were the same price, there would be a clear winner for “the best.”
- ▶ **Examples of Vertical Differentiation:** Branded products vs. generics, A basic black shirt from Hanes vs. a basic black shirt from a top designer, the vehicle makes.

What are Types of Product Differentiation? III

- ▶ **Mixed Differentiation** Also called “simple differentiation,” mixed differentiation refers to differentiation based on a combination of factors. Often, this type of differentiation gets lumped in with horizontal differentiation.
- ▶ **Examples of Mixed Differentiation:** Vehicles of the same class and similar price points from two different manufacturers.

Types of product differentiation

- ▶ Horizontal product differentiation
 - Linear Hotelling model
 - Quadratic Hotelling model
- ▶ Vertical product differentiation
 - Quality choice model

If for equal prices consumers do not agree on which product is the preferred one, products are horizontally differentiated; If on the contrary, for equal prices, all consumers prefer one over the other product, products are vertically differentiated.

Simple location model I

- ▶ product location choice
- ▶ industries which prices are regulated
- ▶ firms only have non-price strategies available.

Simple location model



Model:

- ▶ two firms, location choice $l_i \in [0, 1]$
- ▶ consumers' locations uniformly distributed on $[0, 1]$
- ▶ prefer closeby product
- ▶ linear transportation cost $\tau|x - l_i|$
 - τ : substitutability between product pairs
- ▶ consumers derive utility $v_i(x) = r - \tau|x - l_i| - \bar{p}$
- ▶ Profit for firm i is

$$\pi_i = \begin{cases} (\bar{p} - c)(l_i + l_j)/2, & \text{if } l_i < l_j \\ (\bar{p} - c)/2, & \text{if } l_i = l_j \\ (\bar{p} - c)(1 - (l_i + l_j)/2), & \text{if } l_i > l_j \end{cases}$$

Simple location model



Need to show:

The unique equilibrium is $l_1 = l_2 = 1/2$.

Conclusion

If duopolists choose product locations (but do not set prices), they offer the same products, i.e., they choose not to differentiate their products.

Linear Hotelling Model



- ▶ Consumers have a reservation value r .
- ▶ A consumer of type x is located at some point x on $[0, 1]$
- ▶ Buy up to 1 unit.
- ▶ Firm i located at l_i somewhere, charges price p_i .
- ▶ Linear transportation cost $t(|x - l_i|) = \tau|x - l_i|$
 - transportation cost parameter τ measures the substitutability between any pair of products.
- ▶ Indirect utility $v_i(x) = r - \tau|x - l_i| - p_i$
- ▶ Consumers uniformly distributed on the interval $[0, 1]$, the mass is 1.
- ▶ Firms decide l_i , then simultaneous set price p_i .

Linear Hotelling Model

II

- ▶ Constant production cost c .

We consider the pricing game for the given locations.

Assume $l_1 < l_2$.

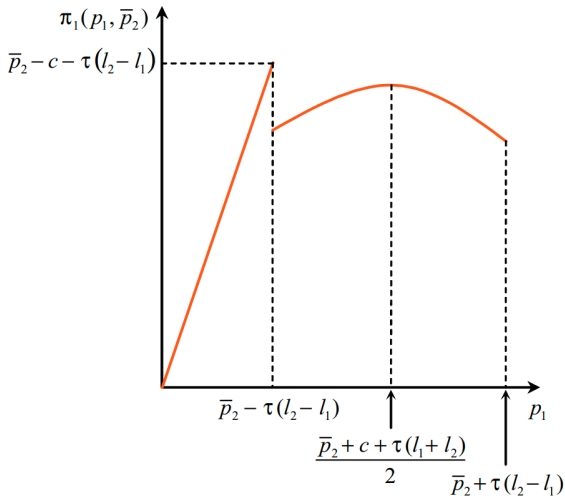
- ▶ Consumer $\hat{x} \in [l_1, l_2]$ who is indifferent between the two products.
 - The indifferent consumer is given by $\hat{x} = \frac{l_1+l_2}{2} - \frac{p_1-p_2}{2\tau}$.

We can then write profits as the price–cost difference times demand:

$$\pi_1(p_1, p_2; l_1, l_2) = \begin{cases} 0 & \text{if } p_1 > p_2 + \tau(l_2 - l_1), \\ (p_1 - c) \left(\frac{l_1+l_2}{2} + \frac{p_2-p_1}{2\tau} \right) & \text{if } |p_1 - p_2| \leq \tau(l_2 - l_1), \\ (p_1 - c) & \text{if } p_1 < p_2 - \tau(l_2 - l_1). \end{cases}$$

Linear Hotelling Model

III



Conclusion

Although product differentiation relaxes price competition, models of imperfect competition in which firms choose product characteristics do not necessarily generate predictions concerning prices and product choices.

Firms may have an incentive to offer better substitutes to generate more demand, which may lead to **instability** in competition.

Quadratic Hotelling Model I

- ▶ Consider that the quadratic function of distance $t(|x - l_i|) = \tau(x - l_i)^2$
- ▶ The indifference condition is then

$$\begin{aligned} p_1 + \tau\hat{x}^2 - 2\tau\hat{x}l_1 + \tau l_1^2 &= p_2 + \tau\hat{x}^2 - 2\tau\hat{x}l_2 + \tau l_2^2 \Leftrightarrow \\ 2\tau\hat{x}(l_2 - l_1) &= \tau(l_2^2 - l_1^2) - (p_1 - p_2) \Leftrightarrow \\ \hat{x}(p_1, p_2) &= \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau(l_2 - l_1)}. \end{aligned}$$

Quadratic Hotelling Model II

- ▶ There exists a unique price equilibrium for each location pair $l_1 \leq l_2$.

$$p_1^*(l_1, l_2) = c + \frac{\tau}{3} (l_2 - l_1) (2 + l_1 + l_2) \text{ and}$$

$$p_2^*(l_1, l_2) = c + \frac{\tau}{3} (l_2 - l_1) (4 - l_1 - l_2).$$

Quadratic Hotelling Model III

- ▶ The payoffs are

$$\hat{\pi}_1(l_1, l_2) = \frac{1}{18} \tau (l_2 - l_1) (2 + l_1 + l_2)^2 \text{ and}$$

$$\hat{\pi}_2(l_1, l_2) = \frac{1}{18} \tau (l_2 - l_1) (4 - l_1 - l_2)^2 .$$

- ▶ In the subgame perfect equilibrium of the two-stage game with quadratic transportation costs, firms choose $l_1 = 0$ and $l_2 = 1$.

General conclusion

Two forces that drives differentiation:

- ▶ competition effect
 - differentiate themselves from competitors to enjoy market power,
 - drives competitor apart
- ▶ market size effect
 - locate where they can best meet consumers' preferences
 - brings competitor together

Quality Choice I

- ▶ Two firms, each produce one product
- ▶ firms chooses quality of product $s_i \in [\underline{s}, \bar{s}]$, and price p_i
- ▶ consumers agree high quality products are better than low quality products

Quality Choice II

- ▶ heterogeneous preference $\theta \in [\underline{\theta}, \bar{\theta}]$,
- ▶ consumers with high θ value quality improvement strongly
- ▶ indirect utility $v_i(p, y, \theta) = \max(0, \max_i(r - p_i + \theta s_i))$
- ▶ Firms profit function:

- $\pi_1(p_1, p_2, s_1, s_2) = \begin{cases} 0 & \text{if } p_1 > p_2 - \underline{\theta}(s_2 - s_1) \\ p_1(\bar{\theta} - \underline{\theta}) & \text{if } p_1 < p_2 - \bar{\theta}(s_2 - s_1) \\ p_1\left(\frac{p_2 - p_1}{s_2 - s_1} - \underline{\theta}\right) & \text{elsewhere} \end{cases}$

Quality Choice III

► Equilibrium:

- $p_1 = \frac{1}{3}(\bar{\theta} - 2\underline{\theta})(s_2 - s_1)$
- $p_2 = \frac{1}{3}(2\bar{\theta} - \underline{\theta})(s_2 - s_1)$

Natural Oligopolies I

Under particular demand characteristics any negligible amount of scale economies is sufficient to make entry profitable for only a limited number of firm.

In the extreme, a natural monopoly may arise even though entry costs are arbitrarily small.

consumer types θ are uniformly distributed on the interval $[\underline{\theta}, \bar{\theta}]$ and obtain utility $\theta s_i - p_i$ from one unit of product i that has quality s_i and is sold at price p_i .

Natural Oligopolies II

- ▶ In the duopoly setting the equilibrium prices are
 - $p_1 = \frac{1}{3}(\bar{\theta} - 2\underline{\theta})(s_2 - s_1)$
 - $p_2 = \frac{1}{3}(2\bar{\theta} - \underline{\theta})(s_2 - s_1)$
- ▶ For both firms to have a positive demand in equilibrium, we must have $\bar{\theta} > 2\underline{\theta}$.
- ▶ If $\bar{\theta} \leq 2\underline{\theta}$, the low-quality firm does not serve any consumers in equilibrium even with 0 price.
- ▶ If there is an entry cost $e > 0$, then the low quality firm has no incentive to enter the market

Conclusion

- ▶ Natural bound for number of firms if vertical product differentiation
- ▶ under particular demand characteristics, any negligible amount of scale economies is sufficient to make entry profitable for only a limited number of firms
- ▶ for horizontal product differentiation, there is always a profitable niche market

Lecture 3:

Differentiation

Jasmine Hao

Logit Model and

Probabilistic

Choice

Empirical Analysis

[Nevo, 2001]

Lecture 3: Product Differentiation - Random Utility Models

Jasmine Hao

Hong Kong University

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Probabilistic choice I

- ▶ Empirical works need assumptions to derive the demand utility. In discrete choice models of product differentiation, it is important to have customers choosing differently (in a particular way) in order to make aggregate demand "smooth".
- ▶ Instead of viewing customers as heterogeneous by nature, one may propose that customers (at least in certain subgroups) are the same before some random variable is realized.
- ▶ After this realization customers are different from each other and heterogeneity results from randomness. As discussed below, the distinction between probabilistic choice and heterogeneous deterministic choice seems to be partly a matter of labelling.

Probabilistic choice II

- ▶ Customers are identical ex ante (statistically identical) but are ex post different due to different realizations of random variables. Modelling customer behaviour as probabilistic is motivated by experimental evidence from the psychology literature. In a controlled experiment, an individual must make repeated choices between two alternatives under "similar" circumstances; sometimes one and sometimes the other will be chosen. Also, intransitivities in choices seem to occur rather frequently.

Probabilistic choice and the logit model

Modelling customer behaviour as probabilistic is motivated by experimental evidence from the psychology literature.

The consumers' random utility is characterized by

$$v_i = r_i - p_i + \epsilon_i.$$

Two explanations for the probabilistic choice model:

- ▶ Individuals behave probabilistically
- ▶ they behave completely deterministically but are observed as if they were acting probabilistically.

Example I

- ▶ In 2007 Apple entered the market for mobile telephones with the iPhone. This new product, which Apple regards as revolutionary, has experienced mixed reactions by consumers. Whereas long queues formed when the product was introduced in the market, other consumers reacted less enthusiastically.
- ▶ Even controlling for differences in observable characteristics, consumers in this market appear to be ex post heterogeneous. Below, we present two complementary explanations for individual behaviour that appears to be probabilistic at the individual level.

Example II

In the words of the psychology literature, individual choice can be seen as the result of the following procedure:

- ▶ a certain stimulus ϵ_i provokes a sensation v_i or a psychological state, which is seen as a realization of a random variable;
- ▶ the response of an individual then depends upon the comparison of sensations.

Interpretation of the probabilistic choice

Example III

- ▶ ϵ_i is interpreted as an uncertainty due to the lack of knowledge available to the observer, which, e.g., is a firm that tries to cater to the tastes of a subpopulation of consumers. Here, ϵ_i takes into account idiosyncratic taste differences inside the subpopulation.
- ▶ In particular, an observer might not be able to describe the state in which a consumer has to take her decision.

Example IV

- ▶ State-dependent utility can be modelled as probabilistic if customers are in different states and the observer does not know the state of the world. Sources of this lack of knowledge can be unobservable characteristics, unobservable variation in individual utilities (e.g. state-dependent utilities where the state is not observable), measurement errors (amount of observable characteristics not perfectly known), and functional misspecification.

Logit Model

- ▶ While a linear model is attractive for computational reasons, such models do not perform well in empirical analysis.
- ▶ A particular non-uniform density that has been applied successfully to empirical problems is to assume that is logistically distributed.
- ▶ The distribution function has the form $F(e) = 1/[1 + \exp\{-e/\mu\}]$.
 - Note that the family of logistic distributions can be parametrized in μ
- ▶ Normalize the mass of consumers to 1 then the probabilistic demand then is of the

$$Q_1 = \frac{1}{1 + \exp\{-(\bar{v} - 1 - \bar{v}_2)/\mu\}} = \frac{\exp\{\bar{v}_1/\mu\}}{\exp\{\bar{v}_1/\mu\} + \exp\{\bar{v}_2/\mu\}}$$

To empirically estimate models of product differentiation (where product characteristics are assumed to be given), we elaborate on the logit model of demand from above to bring it closer to the data. Suppose that consumers can choose between n products in the market and an outside good with the utility \bar{v}_0 normalized to zero.¹⁷ Set for simplicity $\mu = 1$. We then can write market shares as

$$\alpha_i = \frac{\exp\{\bar{v}_i\}}{1 + \sum_{j=1}^n \exp\{\bar{v}_j\}}.$$

All consumers have the same mean utility level \bar{v}_i and this mean utility level is assumed to take the form

$$\bar{v}_i = \beta x_i + \xi_i - \gamma p_i, \tag{5.1}$$

where x_i is the vector of observed product characteristics and β is the corresponding vector of parameters. The variable ξ_i contains the influence of all unobserved characteristics and can be interpreted as the mean utility derived from unobserved characteristics. Finally, γ is the parameter associated to price. Then

$$\log \alpha_i - \log \alpha_0 = \beta x_i + \xi_i - \gamma p_i. \tag{5.2}$$

So far we have only considered the demand side. On the supply side, suppose that firms set prices according to the prediction of a Nash equilibrium in prices. Note that equilibrium prices respond to costs and costs are not directly observable. However, costs are affected by product characteristics, part of which are observable to us. Denote the vector of relevant observable product characteristics on the cost side by w_j . Then we assume that the constant marginal cost of production is of the form $c_i = \kappa w_i + \omega_i$, where ω_i is the mean cost derived from unobservable characteristics and κ is the vector of parameters associated to the observable characteristics. Profits of firm i are $\pi_i = (p_i - c_i)M\alpha_i$. Assuming that a price equilibrium exists and is uniquely determined by the first-order condition, we obtain as the relevant pricing equation,

$$(p_i - c_i) \frac{\partial \alpha_i}{\partial p_i} + \alpha_i = 0.$$

This equation can be rewritten as

$$\begin{aligned} p_i &= c_i + \frac{\alpha_i}{|\partial\alpha_i/\partial p_i|} = \kappa w_i + \frac{\alpha_i}{|\partial\alpha_i/\partial p_i|} + \omega_i \\ &= \kappa w_i + \frac{1}{\gamma} \frac{\alpha_i}{\partial\alpha_i/\partial \bar{v}_i} + \omega_i. \end{aligned}$$

In the logit model, $\partial\alpha_i/\partial \bar{v}_i = \alpha_i(1 - \alpha_i)$ so that the pricing equation becomes

$$p_i = \kappa w_i + \frac{1}{\gamma} \frac{1}{1 - \alpha_i} + \omega_i, \quad (5.3)$$

where parameters κ and γ have to be estimated. If we consider ω_i as an error term, we can jointly estimate Equations (5.2) and (5.3).

While the logit model serves well as a first model to analyse a differentiated product industry, the underlying assumptions and the implied properties make it unsuitable for many empirical implementations.¹⁸ Note that own- and cross-price elasticities of the logit take the form

$$\eta_{ij} = \frac{\partial\alpha_i}{\partial p_j} \frac{p_j}{\alpha_i} = \begin{cases} -\gamma p_i(1 - \alpha_i) & \text{if } i = j, \\ \gamma p_j \alpha_j & \text{otherwise.} \end{cases}$$

RTC Market I



RTC Market II

Beginning of the industry

- ▶ The first ready-to-eat cold breakfast cereal was probably introduced by James Caleb Jackson in 1863, at his Jackson Sanatorium in Dansville, New York.
- ▶ The real origin of the industry, however, was in Battle Creek, Michigan. It was there that Dr. John Harvey Kellogg, the manager of the vegetarian Seventh-Day Adventist health Sanatorium, introduced ready-to-eat cereal as a healthy breakfast alternative.
- ▶ Word of the success of Kellogg's new product spread quickly and attracted many entrants, one of which was Charles William Post, founder of the Post Cereal company.

RTC Market III

- ▶ Additional entrants included Quaker Oats, a company with origins in the hot oatmeal market, a Minneapolis based milling company, later called General Mills, and the National Biscuit Company, now known as Nabisco.

RTC Market IV

High Concentration

- ▶ For economists the concentration of the industry is troublesome because the industry leaders have been consistently earning high profits.
- ▶ This has drawn the attention of regulatory agencies to the practices in the industry. Perhaps the best-known case was the "*shared monopoly*" complaint brought by the FTC against the top three manufacturers - Kellogg, General Mills, and Post - in

RTC Market V

the 1970's.

TABLE I
VOLUME MARKET SHARES

	88Q1	88Q4	89Q4	90Q4	91Q4	92Q4
Kellogg	41.39	39.91	38.49	37.86	37.48	33.70
General Mills	22.04	22.30	23.60	23.82	25.33	26.83
Post	11.80	10.30	9.45	10.96	11.37	11.31
Quaker Oats	9.93	9.00	8.29	7.66	7.00	7.40
Ralston	4.86	6.37	7.65	6.60	5.45	5.18
Nabisco	5.32	6.01	4.46	3.75	2.95	3.11
C3	75.23	72.51	71.54	72.64	74.18	71.84
C6	95.34	93.89	91.94	90.65	89.58	87.53
Private Label	3.33	3.75	4.63	6.29	7.13	7.60

Source: IRI Infoscan Data Base, University of Connecticut, Food Marketing Center.

- ▶ The focus of that specific complaint was one of the industry's key characteristics: an enormous number of brands.

RTC Market VI

- There are currently over 200 brands of RTE cereal, even without counting negligible brands. The brand-level market shares vary from 5% Kellogg's Corn Flakes and General Mills' Cherrios to 1% (the 25th brand) to less than 0.1% (the 100th brand).

[Nevo, 2001] I

- ▶ The ready-to-eat cereal industry is characterized by high concentration, high price-cost margins, large advertising-to-sales ratios, and numerous introductions of new products.
- ▶ Previous researchers have concluded that the ready-to-eat cereal industry is a classic example of an industry with nearly collusive pricing behavior and intense non-price competition.
- ▶ This paper empirically examines this conclusion and attempts to separate these margins into three sources:
 1. product differentiation
 2. multi-product firm pricing
 3. potential price collusion

[Nevo, 2001] II

- ▶ The results suggest that given the demand for different brands of cereal, the first two effects explain most of the observed price-cost margins.
- ▶ The paper concludes that prices in the industry are consistent with noncollusive pricing behavior, despite the high price-cost margins.

Leading firms are able to maintain a portfolio of differentiated products and influence the perceived product quality. It is these two factors that lead to high price-cost margins.

References I



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