Lecture 2: Competition

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Oligopoly

Oligopoly: a small number of firms acting independently but aware of one another's existence.
Assumptions:

- Consumers are price takers.
- All firms produce homogeneous (identical) products: Consumers perceive no differences among them.
- There is no entry into the industry, so the number of firms remains constant over time.
- Firms collectively have market power: They can set price above marginal cost.
- Each firm only sets its price or output (not advertising or other variables).

Bertrand Competition

- Two firms, homogeneous product, same cost c
- $Q_i(p_i) = \begin{cases} Q(p_i) & \text{if } p_i < p_j \\ \alpha_i Q(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$
- a unique pure-strategy equilibrium in which both firms set price equal to marginal costs, $p = c$.
- In the homogeneous product Bertrand duopoly with identical and constant marginal costs, the equilibrium is such that firms set prices equal to marginal costs and thus do not enjoy any market power.
  - Inuition: firms cannot support prices above marginal cost because small price cuts can lead to large increases in quantity demanded and profits.
- What happens if $c_1 < c_2$?

Bertrand Competition - Best Response Function

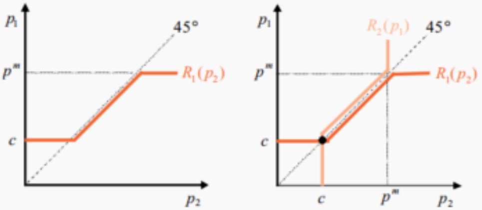

Figure 3.1 Reaction functions and equilibrium in the Bertrand duopoly (with homogeneous product, and identical and constant marginal costs)

Conclusions

In the homogeneous product Bertrand duopoly with identical and constant marginal costs, the equilibrium is such that firms set prices equal to marginal costs and thus do not enjoy any market power.

Price Competition with Uncertain Costs I

- Firms has private information about its marginal costs.
- n firms face the market demand function $Q(p) = 1 - p$ and a marginal cost i.i.d drawn from $Unif[0,1]$.
- $\hat{p}_{-i}$ is the lowest price charged by other players.
- $q_i = \begin{cases} 1 - p_i & \text{if} \quad p_i < \tilde{p}_{-i} \\ (1 - p_i)/m & \text{if} \quad p_i = \tilde{p}_{-i} \\ 0 & \text{if} \quad p_i > \tilde{p}_{-i} \end{cases}.$
- The firm i's expected profit is $(p_i - c_i)\text{Prob}(p_i < \hat{p}_{-i})$.

Price Competition with Uncertain Costs II

The firm's objective function is

$$\max_{p_i} (p_i - c_i)(1 - p_i)[1 - p^{*-1}(p_i)]^{n-1}.$$

The first-order condition of profit maximization is

$$(1 + c_i - 2p_i)[1 - p^{*-1}(p_i)]^{n-1} - (p_i - c_i)(1 - p_i)(n-1)[1 - p^{*-1}(p_i)]^{n-2}\frac{\partial p^{*-1}(p_i)}{\partial p_i} = 0.$$

We rewrite the equation and get

$$p^{*\prime}(c_i)(1 + c_i - 2p^*(c_i))[1 - c_i]^{n-1} - (p^*(c_i) - c_i)(1 - p^*(c_i))(n-1)[1 - c_i]^{n-2} = 0.$$

Dividing by $[1 - c_i]^{n-2}$ and rearranging gives the differential equation

$$p^{*\prime}(c_i) = \frac{(n-1)(p^*(c_i) - c_i)(1 - p^*(c_i))}{(1 - c_i)(1 + c_i - 2p^*(c_i))}.$$

Conclusions

Conclusions:

1. all firms set prices above marginal cost.
2. The price–cost margin is increasing with the efficiency of the firm.
3. In the limit, as n turns to infinity, price converges to marginal costs.
4. In this respect, the model delivers qualitatively similar results as the Cournot model.

Price Competition with Differentiated Product I

- Firms have constant and identical marginal cost.
- wo products (noted 1 and 2) are located at the extreme locations of the $[0,1]$(Hotelling model).
- Firms maximize $\pi_i = (p_i - c)Q_i(p_i, p_j)$.
- A consumer's indirect utility is written as $r - \tau|l_i - x| - p$.
- The demand function $Q_i(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2\tau}$.

Price Competition with Differentiated Product II

Profit functions then become

$$\pi_i = (p_i - c)\left(\frac{1}{2} + \frac{p_j - p_i}{2\tau}\right).$$

The first-order condition of profit maximization is

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{2\tau}\left(p_j - 2p_i + c + \tau\right) = 0.$$

Solving the previous equation for p_i, we derive firm i's reaction function:

$$p_i = \tfrac{1}{2}(p_j + c + \tau),$$

Conclusion If products are more differentiated, firms enjoy more market power.

Extension I

We can extend the analysis to a localized competition with n firms.

- Firms are equidistantly located on a circle with circumference 1 and consumers are uniformly distributed on this circle(Salop model).
- The consumer $\hat{x}_{i,i+1}$ who is indifferent between firms i and $i+1$ is defined by

$$r - \tau\left(\hat{x}_{i,i+1} - \frac{i}{n}\right) - p_i = r - \tau\left(\frac{i+1}{n} - \hat{x}_{i,i+1}\right) - p_{i+1} \text{ or, equivalently,}$$

$$\hat{x}_{i,i+1} = \frac{2i+1}{2n} + \frac{p_{i+1} - p_i}{2\tau}.$$

Extension II

- The demand function

$$Q_i(p_i, p) = \left(\frac{2i+1}{2n} + \frac{p_{i+1} - p_i}{2\tau}\right) - \left(\frac{2i-1}{2n} + \frac{p_i - p_{i-1}}{2\tau}\right)$$
$$= \frac{1}{n} + \frac{p - p_i}{\tau}.$$

- The firm i's maximization problem is

$$\max_{p_i} (p_i - c)\left(\frac{1}{n} + \frac{p - p_i}{\tau}\right).$$

The first-order condition gives $1/n + (p - 2p_i + c)/\tau = 0$. Setting $p_i = p$ yields

$$p = c + \tau/n.$$

Conclusion

Conclusion:

- If products are more differentiated, firms enjoy more market power.
- A larger number of firms leads to closer substitutes on the circle.

Cournot Competition

- Firms choose quantities, price clear the market.
- Total output $q = q_1 + q_2 + ... q_n$.
- Total cost for production $C_i(q_i) = c_i q_i$. (heterogeneous production function)
- The inverse demand function is $P(q) = a - bq$.
- Two firm Cournot equilibrium: $q_1^* = \frac{a-2c_1+c_2}{3b}$, $q_2^* = \frac{a-2c_2+c_1}{3b}$.

Cournot Pricing Formula

maximize $\pi_i = P(q)q_i - C_i(q_i)$

The pricing formula $\frac{P(q)-C_i'(q_i)}{P(q)} = \frac{\alpha_i}{\eta}$, where $\alpha_i = q_i/q$.

- Implications:
  - In the linear Cournot model with homogeneous products, a firm's equilibrium profits increase when the firm becomes relatively more efficient than its rivals.
  - The (symmetric linear) Cournot model converges to perfect competition as the number of firms increases.
  - In the Cournot model, the markup of firm i is larger the larger is the market share of firm i and the less elastic is market demand.
  - In the linear Cournot model with homogeneous products, the Herfindahl index is an appropriate measure of market power since it captures the average markup in equilibrium.

Let us denote $q_{-i} \equiv q - q_i$ the sum of the quantities produced by all firms but firm i. The inverse demand can then be rewritten as

$$P(q_i, q_{-i}) = (a - bq_{-i}) - bq_i \equiv d_i(q_{-i}).$$

Accordingly, firm i will produce a lower quantity. We now confirm this intuition analytically. Firm i chooses q_i to maximize its profits $\pi_i = (a - b(q_i + q_{-i}))q_i - c_i q_i$, which can also be written as $d_i(q_{-i})q_i - c_i q_i$, meaning that firm i acts as a monopolist on its residual demand. The first-order condition of profit maximization is expressed as

$$a - c_i - 2bq_i - bq_{-i} = 0 \tag{3.3}$$

or, solving for q_i, as

$$q_i(q_{-i}) = \tfrac{1}{2b}(a - c_i - bq_{-i}). \tag{3.4}$$

At the Cournot equilibrium, Equation (3.4) is satisfied for each of the n firms. In other words, each firm 'best responds' to the choices of the other firms. Summing the equations (3.4) derived for the n firms, we obtain

$$\sum_{i=1}^{n} q_i = \tfrac{1}{2b}\left(na - \sum_{i=1}^{n} c_i - b\sum_{i=1}^{n} q_{-i}\right).$$

- We find the quantity that firm i produces at the Cournot equilibrium

$$q_i^* = \frac{1}{2b}\left(a - c_i - b\left(\frac{na - C}{b(n+1)} - q_i^*\right)\right) \Leftrightarrow q_i^* = \frac{a - (n+1)c_i + C}{b(n+1)} \Leftrightarrow$$

$$q_i^* = \frac{a - nc_i + C_{-i}}{b(n+1)}. \quad (3.5$$

- We observe that π_i^* decreases with c_i and increases with C_{-i}, which allows us to state the following lesson.

$$\pi_i^* = (P(q^*) - c_i)q_i^* = b(q_i^*)^2 = \frac{(a - nc_i + C_{-i})^2}{b(n+1)^2}. \quad (3.6)$$

Conclusion

- In the linear Cournot model with homogeneous products, a firm's equilibrium profits increase when the firm becomes relatively more efficient than its rivals (i.e., all other things being equal, when its marginal cost decreases or when the marginal cost of any of its rivals increases)

Quantity v.s. Price Choice

- For a given industry, which model?
  - Is price difficult to adjust in the short run?
  - Is quantity difficulty to adjust in the short run?
- Limited capacity
  - airline
  - hotel
- Costly menu adjustment
  - Newspaper

Limited capacity and price competition

Bertrand model presumes that firms can serve any demand at constant marginal costs.

For a large part of industrial production, the assumption of constant (or even decreasing) unit costs may be an appropriate assumption.

However, this only holds as long as capacity is not fully utilized. Increasing output beyond capacity limits is often prohibitively costly so that in the short run, a firm has to respect these capacity choices. This critique of the Bertrand setting was first made by Edgeworth (1897).

Limited capacity I

- two-stage game: in stage 1, firms set capacities q_i simultaneously; in stage 2, firms set prices p_i simultaneously.
- cost of capacity is c and is incurred in the first stage; then, once capacity is installed, the marginal cost of production in the second stage is zero.
- firms are aware that their capacity choice may affect equilibrium prices.

Limited capacity II

- Linear demand $Q(p) = a - p$,
- the profit maximization capacity must $\bar{q}_i = a^2/(4c)$.
- Firm 2 face the residual demand $\hat{Q}(p_2) = Q(p_2) - \bar{q}_1$ if $Q(p_2) - \bar{q}_1 \geq 0$.
- Hence, for $p_1 < p_2$, profits are

$$\begin{cases} \pi_1 = (p_1 - c)\bar{q}_1, \\ \pi_2 = p_2\hat{Q}(p_2) - c\bar{q}_2 = p_2[Q(p_2) - \bar{q}_1] - c\bar{q}_2. \end{cases}$$

- the equilibrium at the second stage of the game is such that both firms set the market-clearing price: $p_1 = p_2 = p^* = a - \bar{q}_1 - \bar{q}_2$.

Conclusion

In the capacity-then-price game with efficient consumer rationing (and with linear demand and constant marginal costs), the chosen capacities are equal to those in a standard Cournot market.

Differentiated Product I

Assume that there is a large number of identical consumers with a linear-quadratic utility function. In particular, suppose that the utility function takes the form

$$U(q_0, q_1, q_2) = aq_1 + aq_2 - \left(bq_1^2 + 2dq_1q_2 + bq_2^2\right)/2 + q_0,$$

This gives rise to the following inverse demand functions

$$\begin{cases} P_1(q_1, q_2) = a - bq_1 - dq_2, \\ P_2(q_1, q_2) = a - dq_1 - bq_2, \end{cases}$$

Differentiated Product II

Let $\tilde{a} = a/(b+d)$, $\tilde{b} = b/(b^2 - d^2)$, and $\tilde{d} = d/(b^2 - d^2)$. Demand functions then take the form

$$\begin{cases} Q_1(p_1, p_2) = \tilde{a} - \tilde{b} p_1 + \tilde{d} p_2, \\ Q_2(p_1, p_2) = \tilde{a} + \tilde{d} p_1 - \tilde{b} p_2, \end{cases}$$

$$p_i^C - p_i^B = \frac{ab}{(2b+d)} - \frac{a\,(b-d)}{(2b-d)} = \frac{ab(2b-d) - a(b-d)(2b+d)}{(2b+d)(2b-d)}$$

$$= \frac{ad^2}{4b^2 - d^2} = \frac{a}{4\left(b^2/d^2\right) - 1} > 0.$$

Price competition always leads to lower prices and larger quantities than quantity competition. Hence, price

Differentiated Product III

as the strategic variable gives rise to a more competitive outcome than quantity as the strategic variable.

Differentiated products: Cournot versus Bertrand I

- Price competition always leads to lower prices and larger quantities than quantity competition.
- Price as the strategic variable gives rise to a more competitive outcome than quantity as the strategic variable.
- have to choose between two different ways in which firms behave in the market place.

Differentiated products: Cournot versus Bertrand II

- The former option (i.e., price competition) appears to be the appropriate choice in case of unlimited capacity or when prices are more difficult to adjust in the short run than quantities. For instance, in the mail-order business, it is costly to print new catalogues or price-lists and, therefore, over some period of time, prices will remain fixed and quantities will adjust accordingly.

Differentiated products: Cournot versus Bertrand III

- In contrast, the latter option (i.e., quantity competition) may be the more appropriate choice in case of limited capacities, even if firms are price setters. A formal explanation of this latter insight has been provided above in the capacity-then-price model, where quantities (seen as capacities) are more difficult to adjust than prices. For instance, this is the case in the package holiday industry: hotel rooms or aircraft seats are usually booked more than one year before a given touristic season and, therefore, prices adjust to sell the available capacities (using, e.g., 'last-minute discounts').

Differentiated products: Cournot versus Bertrand IV

What is the appropriate modelling choice?

- One basic insights of oligopoly theory is that the market outcome under imperfect competition depends on the variable, price or quantity.
- Note first that this question is pointless in a monopoly setting.
- The difference between price and quantity competition materializes in the residual demand a firm faces given the action of the competitor.

Quantity I

- Demand function $P(q_1, q_2) = a - q_1 - q_2$.
- Identical marginal cost $c = 0$.
- Given the quantity q_1, the follower chooses q_2 to maximize $\pi_2 = q_2 P(q_1, q_2)$.
- The leader's program

$$\max_{q_1} \pi_1 = (a - q_1 - q_2(q_1)) q_1 = \frac{1}{2}(a - q_1) q_1.$$

- $q_1^L = a/2$, $q_2^F = a/4$.
- The profit at the subgame perfect equilibrium:

$$\pi_1^L = \frac{a^2}{8} \quad \text{and} \quad \pi_2^F = \frac{a^2}{16}.$$

Quantity II

- Compared to simultaneous quantity choices, the leader produces a larger quantity and makes larger profits, whereas the follower produces a lower quantity and makes lower profits.
- Consider a duopoly producing substitutable products and let one firm (the leader) choose its quantity before the other firm (the follower).
- At the subgame perfect equilibrium of this two-stage game, firms enjoy a first-mover advantage.
- Furthermore, the leader is better off and the follower is worse off than at the Nash equilibrium of the Cournot game (in which firms choose their quantity simultaneously).

Price I

- Suppose $c_1 < c_2$.
  - In simultaneous game, $p = c_2$ is the most reasonable equilibrium.
  - In sequential game, firm 1 choose p_1. The best response of firm 2 is $p_2 = \begin{cases} p_1 - \epsilon & \text{if } p_1 > c_2 \\ c_2 & \text{otherwise} \end{cases}$.
- Let the symmetric demands be given $Q_1(p_1, p_2)$ and $Q_2(p_2, p_1)$,

$$\left.\frac{\partial \pi_1^L}{\partial p_1}\right|_{p_1 = p_1^B} = \underbrace{Q_1\left(p_1^B, p_2^B\right) + p_1^B \frac{\partial Q_1\left(p_1^B, p_2^B\right)}{\partial p_1}}_{=0 \text{ by the FOC of the Bertrand game}} + p_1^B \underbrace{\frac{\partial Q_1\left(p_1^B, p_2^B\right)}{\partial p_2}}_{>0} \underbrace{\frac{dp_2}{dp_1}}_{>0}.$$

Price II

- When the unit costs of the two firms are sufficiently close, the firms have the second-mover advantage.
- For sufficiently different unit costs, it can be shown that only the high-cost firm has a second-mover advantage, whereas the low-cost firm has a first-mover advantage.
- In a sequential price setting game, at least one firm has a second-mover advantage.

Consider a duopoly producing substitutable products and let one firm (the leader) choose its quantity before the other firm (the follower). At the subgame perfect equilibrium of this two-stage game, firms enjoy a first-mover advantage. Furthermore, the leader is better off and the follower is worse off than at the Nash equilibrium of the Cournot game (in which firms choose their quantity simultaneously).

Strategic Substitute and strategic complement I

- variables are strategic substitutes if the reverse inequalities hold. In particular, in the continuous version, strategic substitutability implies that best response functions are downward sloping

Strategic complementarity implies that the best response functions are upward sloping
If the firms' choices are strategic complements (i.e., if best response functions slope upwards) and if an increase in some parameter of the market environment raises marginal profits, then an increase in this parameter leads firms to increase their strategic choice at equilibrium.

Lecture 2: Firms

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Firms

What is a firm?

In this course, simplified representation: given costs and demand, firms choose prices or production quantities to maximize profits.

Later: advertising, and other strategies.

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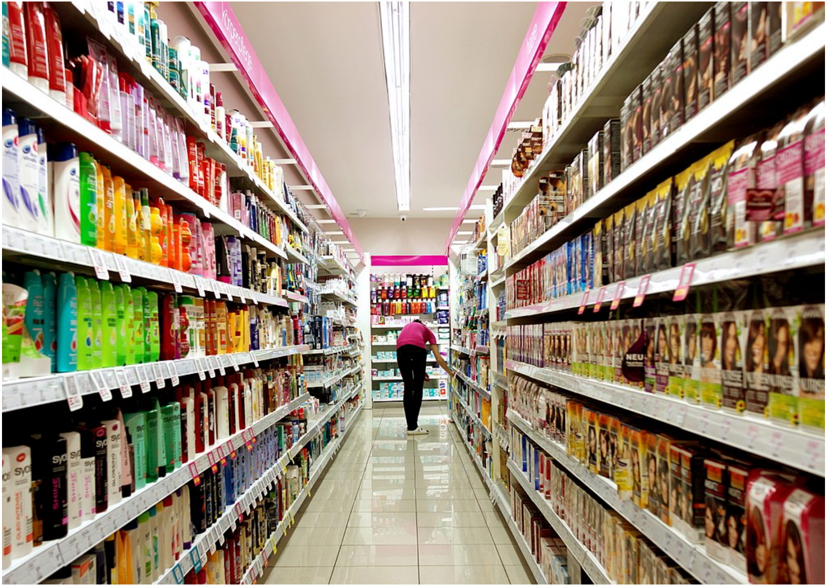

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Road Map

1. Structure, Conduct, Performance
    - SCP Paradigm
    - SCP Elements
2. The Firm's Problem
    - Monopolist
    - Oligopolist

Structure-Conduct-Performance Paradigm

SCP Paradigm

Structure-Conduct-Performance (Bain, 1959; Scherer, 1980.)

- Empirical approach used in the 1950s
- Traditionally, $S \Rightarrow C \Rightarrow P$
  - For example, typical regression: $markup_i = \beta_0 + \beta_1 HHI_i + u_i$

SCP Paradigm

Structure-Conduct-Performance (Bain, 1959; Scherer, 1980.)

- Empirical approach used in the 1950s
- Traditionally, $S \Rightarrow C \Rightarrow P$
  - For example, typical regression: $markup_i = \beta_0 + \beta_1 HHI_i + u_i$
  - Why is this wrong?
- A better model is $S \Leftrightarrow C \Leftrightarrow P$

Market Structure

- Number of firms: Monopoly, oligopoly, firm concentration
- Exports, imports
- Number of sellers: monopsony, ...
- Costs: fixed costs, marginal cost.
- Entry barriers
- Demand

Firms Conduct

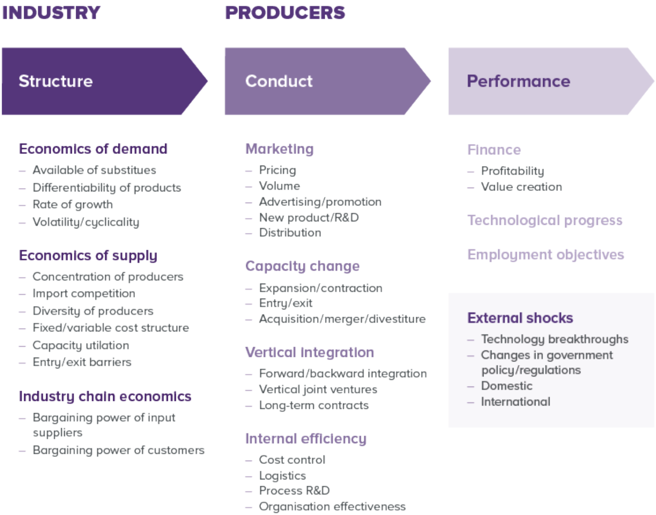
Strategy

- Perfect competition, collusion
- Advertising
- Product variety
- R&D

Firms Performance

- Profits
- Markups
- Margins

Figure: SCP Paradigm



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The Firm’s Problem

Firm's Problem

How do firms take their pricing, production decisions?

We assume firms maximize profits (also, minimize costs)

Steps:

- Write down profit function
- Maximize wrt prices (Bertrand) or quantities (Cournot)
- Find optimal price

Monopolist's Problem (1)

$$\pi = (p - c)q(p) - F_i$$

- Price is strategic variable (for simplicity)
- Cost is constant
- What are costs?
  - Opportunity costs
  - Economic costs
  - Marginal cost
  - Economies of scale $C'(q) < 0$ (decreasing average cost)

Monopolist’s Problem (2)

$$\max_p \pi = (p - c)q(p) - F_i$$

Monopolist's Problem (2)

$$\max_p \pi = (p-c)q(p) - F_i$$

$$\text{FOC: } (p-c)q'(p) + q(p) = 0$$

$$p - c = -\frac{q(p)}{q'(p)}$$

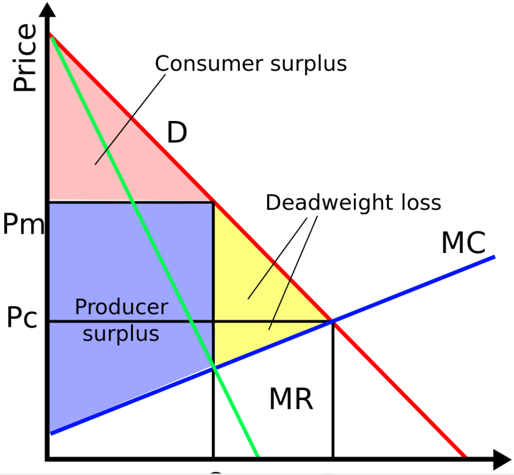
$$\frac{p-c}{p} = -\frac{1}{\eta}$$

where $\eta = q'(p)\frac{p}{q(p)}$ is the demand elasticity.

- $\frac{p-c}{p}$ is the Lerner index.
- The more inelastic the demand, the higher the markup
- We call $\frac{p-c}{p} = -\frac{1}{\eta}$ the inverse elasticity rule.
- What happens if $|\eta| < 1$?

Monopolist's Problem (3)

A monopoly maximizes profits by producing where marginal revenue equals marginal costs.



Monopolist's Problem (4)

$$\frac{p-c}{p} = -\frac{1}{\eta}$$

- What are the differences between monopoly and perfect competition?
- What if cost changes with p or q?
- Costs are unobserved! How do we know markups?

Oligopolist's Problem (1)

Bertrand Competition (with differentiated products)

$$\pi_i = (p_i - c_i)q(\cdot) - F_i$$

- Price competition
- Assume other firms' prices are constant (why?)
- For simplicity, cost is constant
- What is product differentiation?

Oligopolist's Problem: Example

Suppose there are two firms that compete on prices, zero marginal cost, and that the demand is

$$q_i = 1 - p_i + 0.5p_j$$

What is the equilibrium market price?

What happens if product differentiation increases?

Oligopolist's Problem (2)

Suppose two firms i and j that compete on prices.

$$\max_{p_i} (p_i - c) q_i(p_i, p_j)$$

$$\text{FOC: } (p_i - c) q'(p_i, p_j) + q_i(p_i, p_j) = 0$$

$$p_i - c = -\frac{q_i(\cdot)}{q_i'(\cdot)}$$

$$\frac{p_i - c}{p_i} = -\frac{1}{\eta}$$

- $\frac{p_i - c}{p_i} = -\frac{1}{\eta}$: inverse elasticity rule again.
- Why?

Strategic Complements

What should firm i do if firm j increases its price

Write p_1 as a function of p_2.

Strategic Complements

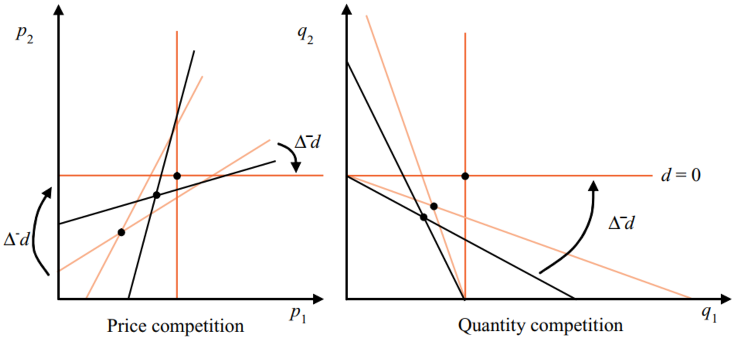


Figure 3.6 Reaction functions for price vs. quantity competition (when firms produce substitutable goods)

Figure: Reaction curves-strategic complements

Quantity Competition

Cournot Competition

$$q_i = 1 - p_i + 0.5p_j$$

What are the equilibrium market quantities?

Prices or Quantities?

What is the strategic variable of the firms?

Prices: retail

Quantities: cement, potash.