Building Trust: A Dynamic Game of Collusive Price Leadership *

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November 7, 2023

Abstract

This paper introduces a dynamic game that explores the challenges faced by firms when initiating collusion. Utilizing a case study of a price-fixing cartel in the Chilean pharmaceutical retail sector, the model addresses the incentive and coordination problems that emerge. Initially, firms colluded in selected markets before expanding their collusion to additional ones. The argument posits that firms increase trust by learning from early collusive outcomes, thereby facilitating subsequent collusion. The paper evaluates various counterfactual antitrust policies and assesses their efficacy in preventing cartels. This paper emphasizes the importance of addressing coordination issues and proposes that failing to account for these issues can lead to misleading policy predictions.

Keywords: Collusion, Dynamic Game, Oligopolistic Price Competition, Firms' Beliefs, Repeated Games.

JEL codes: C72, C73, D43, L13, L41

*The author is very grateful to Hiro Kasahara, Florian Hoffman, Victor Aguirregabiria, and Paul Schrimpf. I also acknowledge Vadim Marmer, Kevin Song, Sam Hwang, Kevin Milligan, Wei Li, Limin Fang, Eduardo Souza-Rodrigues, and conference and seminar participants for helpful comments. I am also grateful for the Gambling Award Fellowship for financial support.

1 Introduction

Collusion has been a central subject in industrial organization ever since the seminal work by Bain (1959). It is detrimental to consumer welfare and threatens fair market competition. As Green et al. (2014) states, the rich literature on collusion often focuses on the implementation phase (see, for example, Harrington Jr and Chang, 2009, 2015; Igami and Sugaya, 2021), with the initiation phase frequently overlooked. There are a few exceptions, such as the work of Byrne and De Roos (2019), which documents empirical evidence of firms learning to coordinate for tacit collusion.

Firms in oligopolistic markets attempting to achieve collusive outcomes, whether in communication or not, face two key problems: the *incentive problem* and the *coordination problem* (Harrington, 2018). The *incentive problem* refers to the need for firms to ensure collusion is profitable enough to warrant participation. The *coordination problem*, on the other hand, is the result of multiple possible outcomes or equilibria in oligopolistic markets, causing uncertainty among firms. To model the coordination problem effectively, it's vital to incorporate firms' 'higher-order knowledge'—their understanding of other firms' mental states, including intentions and beliefs (Aumann and Brandenburger, 1995; Green et al., 2014). Few previous empirical works explicitly account for the coordination problem in the initiation stage of cartel formation.

This research presents a dynamic game model under incomplete information, expanding on the widely applied Markov perfect equilibrium (MPE) framework of Ericson and Pakes (1995) and Bajari et al. (2007). In a novel approach, the model allows for firms' beliefs to be conditional on variables that don't directly affect payoffs, suggesting that firms' perceptions of their competitors' actions may not always correspond to the "true" values. The concept of "biased belief functions" is introduced, which evolve according to the history of the game, thereby capturing trust-building during the initiation of collusion. This model deviates from the fundamental specification proposed by Maskin and Tirole (1987). While Maskin and Tirole (1987)'s framework focuses on the equilibrium where firms' strategies are solely functions of payoff-relevant state variables, my model expands this view. While ensuring computational feasibility for estimation, we avoid limiting ourselves to a large class of models by insisting on rational beliefs for firms. I argue that during the build-up of collusion, firms' beliefs may not always be rational and may involve learning about other firms' behaviour. Hence, I introduce a new equilibrium concept, the Trust Building Equilibrium (TBE), which provides an alternative to MPE in dynamic games. The TBE allows for more complex pricing dynamics due to its flexibility and incorporates the MPE solution. I apply this model to analyze the prosecuted price-fixing cartel involving Chile's three major pharmacy chains—Salcobrand, FASA, and Cruz Verde—in 2008. This case, examined by Alé Chilet (2016, 2018) and detailed in an expert report by Núñez et al. (2008). The firms engaged in collusive price leadership (Rotemberg and Saloner, 1990). According to court documents, firms started by colluding on a few prescription drugs and then expanded the scope of price-fixing after successful collusion. The concept of trust in this paper diverges from the view advanced by Alé Chilet (2016). This paper claims that firms start collusion with "safer" products where price leadership costs are smaller, gradually moving onto "riskier" products. They describe this decreasing preference for safer markets over time as "trust building." In contrast, I propose that the initial incentive to collude is predominantly stronger in safer markets, but this differential narrows over time. As firms become increasingly adept at collusion, their propensity to collude increases across all markets, although safer ones continue to offer a slightly higher appeal.

This study extends the results of Aguirregabiria and Magesan (2020) by identifying beliefs as a non-payoff relevant variable, serving as an exclusion restriction condition. With the TBE framework, my analysis reveals an increasing propensity of firms to initiate a price increase over time, thereby promoting collusion across multiple products. In contrast, the standard MPE framework predicts immediate collusion across all products and fails to consider the leader's evolving incentives over time.

I employ two counterfactual experiments: an *adjustment friction* scenario and a *divestiture* scenario. In the *adjustment friction* scenario, I assume that firms encounter increased menu costs when changing prices. This added friction creates a more complex inter-firm dynamic. The leading firm that initiates the collusive price increase may anticipate reluctance from followers to adjust their prices accordinglly. On the other hand, the *divestiture* scenario is inspired by Harrington Jr

(2018). [1] Interestingly, the two distinct equilibrium concepts, TBE and MPE, offer divergent recommendations in these scenarios. The TBE suggests a preference for the divestiture approach, while the MPE prefers the adjustment friction strategy. The outcomes of these experiments underscore the necessity of accurately modelling the coordination problem. This becomes particularly crucial when I evaluate the impact of antitrust policies on the initiation of collusion. These findings underscore the need for policymakers to consider coordination dynamics when devising strategies to deter collusion.

This paper contributes to the literature in two ways. First, it introduces a dynamic pricing game model for collusion initiation that incorporates 'higher-order knowledge' and extends belief identification as per Aguirregabiria and Magesan (2020). Second, it presents empirical evidence indicating that successful collusion increases the propensity for future collusion, primarily due to heightened expectations of adherence to the collusive equilibrium.

Related Literature

This model introduces a non-payoff-relevant, belief-altering signal that facilitates the coordination of firms in transitioning from a non-collusive to a collusive focal point. The non-payoff-relevant state variable can serve as a coordination device among multiple equilibria.[2] The paper also links multi-market contact with the exercise of market power through firms' learning: firms initially coordinate in some markets and subsequently apply this conduct in others.[3]

This study engages with several recent related works in cartel formation. Firstly, it aligns with Calvano et al. (2020), which exhibits that independently created algorithms can unintentionally learn to collude via Q-learning, a process involving updates to the choice-dependent value function. These value functions are consid-

[1] I simulate a situation where the government enforces a 25% asset divestiture by each chain store, ultimately leading to a new fourth chain store.

[2] This model addresses the coordination problem as a focal point issue. Agents with arbitrary beliefs choose rationalizable actions, certain about common knowledge facts. Firms' beliefs assign a high prior probability to a specific profit-maximizing equilibrium being played (Green et al., 2014, pp. 28). This setup can be applied to other collusion cases like bid-rigging cartels (Green and Porter 2014, pp. 31) and examples in Knittel and Stango (2003) and Lewis (2015).

[3] Empirical findings suggest that multi-market contact facilitates collusion (see Evans and Kessides (1994); Parker and Röller (1997); Ciliberto and Williams (2014)).

ered distinct components: beliefs about other firms' actions and payoff functions. In this context, firms update their beliefs while the payoff functions remain unchanged. Secondly, the paper connects to Byrne and De Roos (2019), which documents the dynamics of learning to coordinate in the retail gasoline market. The study scrutinizes how firms' interactions in pricing behaviour gradually lead to coordinated behaviour over time. This model addresses the coordination problem by allowing firms' beliefs to diverge from other firms' policy functions and enabling bias reduction through frequent interactions. Lastly, my research relates to the empirical literature on collusion, especially Miller et al. (2021), which investigates firms' incentives for collusive price leadership. My study sets itself apart by focusing on the temporal evolution of coordination problems, specifically in instances where firms have successfully implemented the price leadership mechanism.

This paper is related to studies on firms forming biased beliefs about model primitives, such as demand, rivals' costs, and strategic behaviour.[4] I explore the concept of modelling strategic uncertainty (Morris and Shin, 2002) with multiple equilibria and higher-order beliefs. Unlike existing research that focuses on static empirical models, such as Cognitive Hierarchy models and Level-K Rationality (see Brown et al., 2013; Ciliberto and Tamer, 2009; Goldfarb and Xiao, 2016; Hortaçsu et al., 2019), I introduce a dynamic framework that addresses strategic uncertainty through biased beliefs. I employ the equilibrium restriction technique to estimate structural parameters, which retains identification power even with multiple equilibria Aguirregabiria and Mira (2007). In contrast to learning models, the paper emphasizes the non-parametric identification of belief evolution and proposes a novel structure for firms' information acquisition.

The following of the paper is structured as follows: Section 2 provides background on the Chilean pharmacy industry, describing the industry background and the collusion case. Section 3 presents a structural model incorporating non-equilibrium beliefs to capture trust building during the cartel formation. Section 4 discusses the identification of model parameters and beliefs from data and includes Monte Carlo simulations for validation. Section 5 reports the main estimation results and findings from counterfactual experiments that examine how outcomes would differ under alternative assumptions about firms' beliefs.

[4]See Borkovsky et al. (2015), Ching et al. (2017), and Aguirregabiria and Jeon (2020) for extensive reviews.

2 Background and Data

This section provides an overview of the Chilean retail pharmacy market from 2006 to 2008.[5] In 2008, the retail drug market in Chile was dominated by three pharmacy chains with a national presence - Cruz Verde, FASA, and Salcobrand - which together held 92% of the market. Of the three, Cruz Verde was the largest with 512 stores, while FASA and Salcobrand had 347 and 295 stores respectively. The remaining 8% of the market was controlled by independent drugstores and small chains, which mainly focused on generic drugs.[6]

Up until 2008, medication prices in Chile were not regulated and there were no controls in place. Additionally, Chile's healthcare system did not frequently reimburse drug expenditures, and branded drugs were typically sold at a premium price. As a result, the prices listed in the dataset reflect the out-of-pocket cost for consumers. Medications could only be purchased at drugstores and not hospitals. Furthermore, doctors would prescribe drugs by their brand name instead of by their molecule name. Additionally, prescription switching was prohibited, even if the same molecule was available under a different brand for prescription-only drugs. When it comes to the price setting process, retail chains would set prices nationally, but prices may vary slightly in different store locations.

2.1 Data

I use pharmacies' proprietary data given to the Competition Authority of Chile. These data include purchase information (transaction prices and quantities) from the three drugstore chains regarding the 222 products on which the chains were accused of a price-fixing cartel for 2006–2008. I calculate the daily nationwide revenue-weighted average price from the data and each chain's daily units sold nationwide. I use various online databases to enhance the dataset, such as Catalog.md, Farmazon.cl and Drugbank.com. Catalog.md provides details about the active ingredients of drugs and lists all the producers who manufacture the drug at the brand level. Farmazon.cl offers prescription information for drugs in Chile,

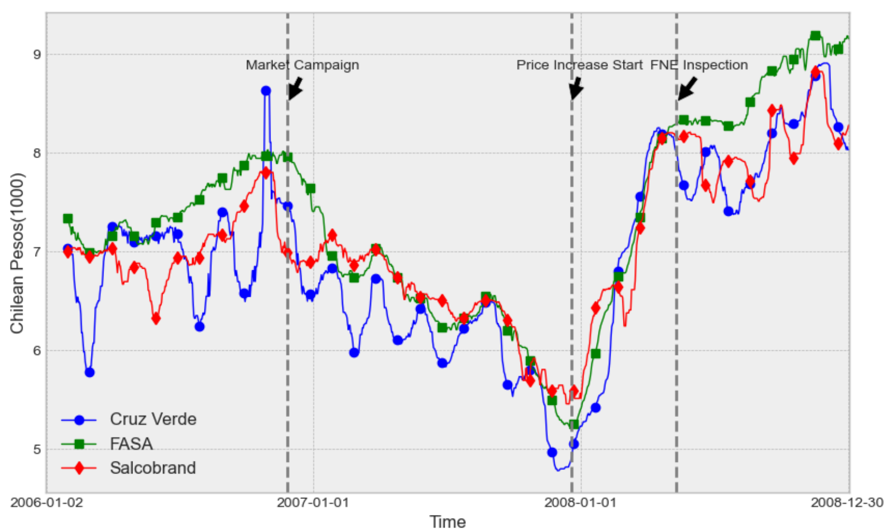
[5]For more information, refer to Alé Chilet (2016, 2018) and the collusion case reports written by the National Economic Prosecutor (NEP) of Chile (Núñez et al., 2008).

[6]These numbers are from the Investors Conference presentation held in December 2008. The presentation was made by FASA in March 2009 and accessed online in July 2012.

whereas Drugbank.com contains treatment-related information for specific molecules.

2.2 Competitive and Collusive Periods

Figure 1: Revenue-weighted Average Drug Prices From 2006 to 2008



Data source: Competition Authority of Chile.
I plot each pharmacy's rolling 7-day average revenue-weighted average prices. For the weighted wholesale prices, I use the total revenue of the three pharmacies as the weight.

The period between 2006 and 2008 can be divided into four distinct phases of price movements, as per a comprehensive case review conducted by Alé Chilet (2016, 2018), the judgement passed by the Tribunal de Defensa de la Libre Competencia (TDLC) of Chile, and the expert report presented by Salcobrand. The revenue-weighted average prices of the 222 drugs are illustrated in Figure 1.

From January to November 2006, the market exhibited traits of intense competition with minimal price fluctuations. Firms offered promotional discounts while maintaining stable prices. The absence of complaints against these firms prior to 2006 suggests that they were competing fairly using similar pricing strategies.

The second phase, referred to as a *price war*, spanned November 2006 to November 2007, during which there were both sharp and gradual reductions in drug prices. The average price decrease was 27%, resulting in prices below wholesale levels. In August 2007, the acquisition of Salcobrand by a prominent local business group appears to have stimulated the drive to lead the coordination among firms.[7]

The third phase, known as the *coordination period*, commenced in December 2007. During this phase, Cruz Verde's advertising campaign was discontinued due to a court ruling on unfair competition, and there was a change in Salcobrand's management. Pharmacies then coordinated their actions, leading to substantial price increases on over 200 products, primarily top-selling, brand-name, prescription-only medications. By early May 2008, this resulted in an average price increase of 50% , coinciding with the antitrust investigation's start. It's important to note that these price hikes were not caused by changes in wholesale prices, as documented by Alé Chilet (2016). [8] During the coordination period, the average price had increased by 50% compared to the pre-coordination period, specifically in October 2007. Crucially, the price increases were not driven by changes in wholesale prices, as stated in the expert report. The coordination period ended in May 2008, leading to an antitrust investigation. However, drug prices remained high. Testimony from a former Cruz Verde board member highlighted that Salcobrand's new management aimed to change the aggressive competition culture among firms.[9]

This paper examines the role of the coordination period in transitioning from non-collusive to collusive equilibria, using a case study to provide insights into the initiation of collusion. It is feasible to expand the model to include other pricing behaviours during different periods, however, doing so would make the model more complex and increase difficulties in estimation.

[7]In April 2007, Empresas Juan Yarur SA (EJY), the parent company of BCI Bank, purchased Salcobrand SA for $130 million. Salcobrand SA, a chain of pharmacies with 230 locations and over 3,000 employees, is legally owned by EJY Dos Ltda., an investment company within EJY's internal structure. Data source: Pharmacies Report, Cuadernos of Investigación.

[8]In 2012, the TDLC found that three pharmacy chains had violated competition rules by engaging in price-fixing activities for certain medicines. These drugs, produced by 37 pharmaceutical companies, belonged to 36 therapeutic categories. The targeted medications included ”éticos” (prescription drugs for severe or chronic conditions) and ”notorios” (non-prescription, high-volume drugs commonly marketed directly to consumers).

[9]The former Cruz Verde board member Fernando Suarez Laureda stated, “Salcobrand's [new management] came to change this dynamic (. . .) of big emotional aggressiveness between the companies because, in fact, Salcobrand present[ed] itself as a neutral competitor that [made] its decisions mostly based on economic principles (. . .).” (Deposition of Fernando Suárez Laureda, p. 224).

2.3 Collusive Price Leadership

Coordinated price increases start with one firm initiating the increase. The first firm to increase the price for a given product is considered the price leader. During the coordination phase, price leader pharmacy managers intentionally increased the prices of specific drugs regularly. The followers would then match these prices within days. Salcobrand is known to be the leader in more than 90% of these price increases. The average retail price of drugs increased gradually because of the regular coordination price increases. Alé Chilet (2016) documented this and the case report by Núñez et al. (2008). [10]

Figure 2 demonstrates the price increase mechanism for the product *Folisanin 5 mg. (30 Pills/Box)*, showing the price before and after the coordinated price increase. This pattern is observed for all products the pharmacies colluded on, with varying start times.

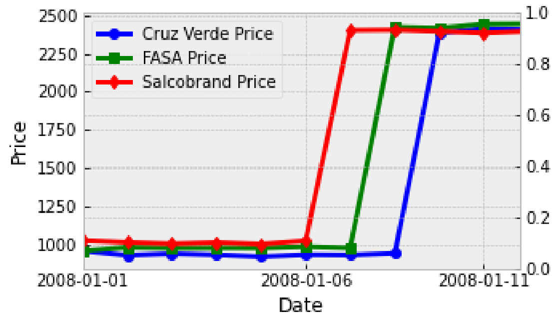
I analyzed the frequency of coordination occurrences during the period and detected patterns. Table 1 displays the frequency of a leader initiating price increases. Before the coordination period, there were 33 instances (1.43 per month) of a leader starting price increases. During the coordination period, this number increased substantially to 208 cases (23.83 per month). After the coordination episode, there were 32 instances (4.57 per month), suggesting that firms retained some coordination memory even after the coordination episode. During the coordination episode, several coordination incidents occurred, which had a lasting impact on the firms' behavior.

2.4 Varying Incentives to Collude

In this section, I provide evidence that firms which have colluded successfully using price leadership in the past are likely to repeat this behaviour. I support this claim by referring to court documents, such as a statement by Salcobrand's business manager. According to the statement, the business manager informed the company's CFO of their plan to raise prices first every week, allowing other chains

[10] After analyzing the data, I did not find any evidence of multiple price levels during the coordination period. Generally, prices tend to increase significantly, and firms closely align their prices before and after a coordinated price increase. Based on this observation, it appears that my assumption of binary prices is a reasonable one for this particular case.

Figure 2: Coordinated Price Increase for *Folisanin 5 mg. (30 Pills/Box)*



The price are quantity-weighted sales price across nationwide sales for the three chains. Data source: Competition Authority of Chile.

Table 1: Frequency of Leaders' Performance During Different Periods

	Before Coordination *Jan 2006 - Nov 2007*	During Coordination *Dec 2007 - May 2008*	After Coordination *July 2008 - Dec 2008*	Total
Price Leader				
Cruz Verde	10	9	12	31
FASA	12	8	10	30
Salcobrand	11	126	10	147
Total	33	143	32	208
	1.43 per month	23.83 per month	4.57 per month	

The term "coordinated price increase" describes a situation that meets the following conditions: (1) The price of a specific product is increased by at least 15% of its average price as of January 2006 or by more than 1500 pesos, and three firms do this. (2) One firm starts the price increase, and the other two follow within four days. (3) The price levels before and after the increase are close, i.e., within a 15% difference. (4) The firms maintain the increased price level for at least three days.

to follow suit a few days later. The business manager expressed a desire to use this strategy for more products and with additional pharmaceutical companies due to the positive results obtained in the past (Observations of the evidence, December 19th, 2007, p.18).

The Cox survival model reported in Table 2 helps to identify the markets in which firms are more likely to collude, and how these incentives change over time. The model uses regression coefficients to show the increase in hazard ratio, where a positive value indicates that collusion occurred earlier in those markets or products. [11] Columns 1 to 2 suggest that firms tend to initiate collusion on products

[11]The survival analysis results in Table 2 are compared to a similar analysis conducted by

Table 2: Timing of Collusion: Market Characteristics

	Proportional Hazard Model			Time-varying Hazard Model		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Cross Elasticity	-0.354	-0.351	-0.3427	-1.0608	-1.3806	-1.3781
	(0.2572)	(0.2481)	(0.2469)	(6.117)	(6.4657)	(6.4683)
Ln Market Size	0.4516***	0.4538***	0.4536***	1.2521	1.4032	1.4059
	(0.0771)	(0.133)	(0.1319)	(2.3947)	(2.719)	(2.7302)
Price Dispersion	-	0.1195	0.118	-	-0.1784	-0.1763
	-	(0.1169)	(0.1189)	-	(1.3273)	(1.3239)
Share Dispersion	-	-0.3425	-0.2112	-	12.4508	12.4217
	-	(3.3214)	(3.3091)	-	(22.4463)	(22.4248)
Market Share Leader	-	-4.7582***	-4.5848**	-	-7.8503	-7.9218
	-	(1.8229)	(1.8172)	-	(18.6303)	(18.7535)
Cross Elasticity * $\log(T)$	-	-	-	0.2045	0.2737	0.2732
	-	-	-	(1.2488)	(1.3288)	(1.3293)
Ln Market Size * $\log(T)$	-	-	-	-0.2472	-0.2785	-0.2791
	-	-	-	(0.4859)	(0.5514)	(0.5539)
Price Dispersion * $\log(T)$	-	-	-	-	0.0368	0.0363
	-	-	-	-	(0.2801)	(0.2794)
Share Dispersion * $\log(T)$	-	-	-	-	-2.4906	-2.4843
	-	-	-	-	(4.7433)	(4.7401)
Market Share Leader * $\log(T)$	-	-	-	-	1.6039	1.6193
	-	-	-	-	(3.9032)	(3.9316)
Succeed Ratio	-	-	0.9193***	-	-	0.0109
	-	-	(0.0871)	-	-	(0.0872)
Total Coord Attemp	-	-	-0.0084***	-	-	0.0002
	-	-	(0.0021)	-	-	(0.0021)
AIC	70949.5716	70206.827	70060.37	69750.8864	69534.101	69537.028
Concordance Index	0.5088	0.4912	0.548	0.9749	0.9749	0.9567
N	22398	22376	21984	22398	22376	21984
log-likelihood	-35472.7858	-35098.4135	-35023.185	-34871.4432	-34757.0505	-34756.514

Market Size is defined as the median of the aggregate quantity sold across all products and all three chains during the base period, which is October 2007.
Share Dispersion represents market share asymmetry. It is computed using market shares from the base period.
Price Dispersion is a measure of price volatility. It is calculated as the mean of weekly price volatility during the base period.
Cross Elasticities are calculated using product-specific demand price coefficients, as detailed in Section 3.2.
The term 'Market Share Leader' represents the firm that has the highest market share in the product where collusion occurred. This collusion is considered to have started on November 1st, 2007, and the variable 'T' refers to the number of days that have passed since then.
In the context of pricing, the "Success Ratio" is the percentage of successful price increases that occurred in the past two weeks, while "Total Coord Attempt" is the total number of price increase attempts made by any of the three firms. An attempt is considered unsuccessful if a firm tries to increase the price, but the other two firms don't follow suit. In such a case, the initiating firm reverts to the original price within five days.
$^{*}p < .1$ denotes statistical significance at the 10% level. $^{**}p < .05$ denotes statistical significance at the 5% level.
$^{***}p < .01$ denotes statistical significance at the 1% level.

with smaller cross-elasticities earlier, although this effect is not statistically significant. This is likely because small price increases do not significantly affect market

Alé Chilet (2016). Their study found that the leader's market share impacts initial collusion, but market size and cross elasticity do not. My study uses a similar set of regressors, including cross elasticity estimations (see column 3 for comparison). My approach differs from that of Alé Chilet (2016) in considering daily coordination instead of weekly coordination for the duration value. Daily coordination allows for a more detailed view of the sequence of events, while the weekly approach may oversimplify it. Additionally, the limited number of weeks available for analysis may affect the reliability of the findings.

share or consumer attention. The size of a market greatly affects the likelihood of collusion, with firms being more willing to collude in larger markets. This could be because it's easier to detect instances of collusion in larger markets, due to the increased visibility of price changes. Market asymmetry can also play a role in collusion, where the market share of the price leader is a crucial factor in determining strategy. The lower the market share for the price leader, the earlier they are willing to lead a price increase. In Column 3, I introduce two new variables: the total attempts at coordination and the success ratio over the preceding two-week period. The success ratio serves as a proxy for a firm's capacity to learn from past interactions. The positive sign indicates recent successes make it easier for firms to engage in successive collusion. The coefficients for cross elasticity, log market size, price dispersion, and leader market share remain largely unchanged with the addition of the success ratio. This suggests that a history of successful coordination consistently enhances firms' incentive to collude, regardless of these other factors. Columns 4 to 6 introduce interaction terms with $\log(t)$, considering firms' changing incentives over time. The signs of the interaction term with $\log(t)$ typically oppose those of the original regressor. This pattern supports Alé Chilet (2016) argument that firms are more likely to collude on products perceived as risky over time. The result suggests that initially, leaders might favour markets with lower cross elasticity, larger market size, and where they have smaller market share. However, as time progresses, these market preferences diminish significantly.

While the available data does not allow us to definitively reject competing theories, such as firms testing market demand reactions, I argue that firms operate in a stable market environment. As such, it is improbable that they are still in the process of learning about market demand. Despite a leadership transition at Salcobrand, the new management's focus remains primarily on pricing strategies, without a substantial emphasis on forecasting. As per Alé Chilet (2016), FASA's statement suggests that Salcobrand's change in ownership has led to a more aggressive market atmosphere. Therefore, this intensified environment, or the firms' perceptions regarding equilibrium selection, serve as the primary driving forces. A report by Vasallo (2010) found that drug prices in Chile diverge from those in other South American countries, suggesting that demand-side shocks may be of lesser importance. The consistent exchange rate in Chile during this period makes it unlikely that a supply-side shock, due to higher exchange rates when purchasing drugs in bulk from pharmaceutical firms, could be a significant factor.

3 Dynamic Pricing Model

This section presents a structural model designed to encapsulate firms' trust-building processes while adopting price leadership. The framework enhances the MPE (Markov Perfect Equilibrium) model (see, e.g., Ericson and Pakes, 1995; Doraszelski and Pakes, 2007) by enabling firms' policy functions to be conditional on a public non-payoff signal, not just the state relevant to payoff.

Lagged prices are relevant to payoff because firms incur menu costs to adjust price levels. Upon reviewing past pricing behaviour, a firm can interpret its chosen price level as either assuming the role of a price leader following a price increase, or maintaining its position within a collusive relationship. The previous successful collusion acts as the non-payoff signal. Given the nature of prescription drugs, successful prior collusion in a different market does not alter the flow payoff. The proposed framework supports more complex interaction patterns by allowing for changing beliefs throughout the game's history.

3.1 Setup of the model

The industry comprises a fixed set of firms, denoted as $i \in \{CV, FA, SB\}$. Each firm offers multiple products, and prices are set simultaneously for each product, represented by $m \in \{1, \ldots, M\}$. In the empirical application of the model, a market is defined by a specific combination of brand and dosage. For instance, "Maltofer Gts. Frasco 30 ml" and "Maltofer 100 mg" are considered two separate markets in the industry even though both products are used to treat iron deficiency but differ in packaging and dosage. Importantly, no substitution occurs between two goods since sales occur within a single day, and my research focuses on customers' immediate decisions. Should a consumer wish to switch brands or dosages, they must obtain a new prescription. On any given day, the consumer has the choice to either purchase from one of the three firms or not purchase at all.

At each period t, all firms observe the current pricing state $\mathbf{x_{mt}} = \mathbf{a_{mt-1}} =$

(a_{imt}, $\forall i$) and choose one price from the choice set $\mathcal{A} = \{0, 1\}$ for every market m. Let $a_{imt} = 1$ denote that firm i has set the price for market m to the collusive level, and $a_{imt} = 0$ denote the competitive level. [12] Firms decide on their pricing strategies with the aim of maximizing their expected discounted payoffs: $\sum_{s=1}^{\infty} \mathbb{E}_t \sum_m \Pi_{imt}$. Here, $\beta \in (0, 1)$ is the discount factor, $\mathbb{E}_t$ is the expectation operator at time t, and Π_{imt} is the firm's payoff at period t on market m.

When a firm decides to become a price leader, it weighs the benefits and costs involved. Price leaders can potentially gain collusive profits in the future, but the position can also be costly due to the potential forgone profits to other firms. The firm that takes on the position of a price leader may harm its reputation among customers, and it may lose on additional sales of non-prescription drugs as customers who visit the store usually buy other items. These factors increase the overall cost of being a price leader, and hence, it is essential to consider them in the firms' strategic decision-making process.

To capture the relative costs, I use two elements: a biased belief parameter and leadership costs. The biased belief parameter determines how much the leader underestimates the followers' incentive to follow. As explained in Section 2.4, the probability of a firm assuming a leadership position increases with the increase in h_t, where h_t is a metric that represents past successful collusion across all markets. However, this increase is not attributed to a higher flow payoff benefit. Instead, it is due to the firm's belief that followers are more likely to comply. The variation in h_t is then used to identify the beliefs.

3.2 Flow Payoffs

Focusing on firms' interactions on a specific market m, I assume that the firms observe a time-evolving common knowledge state variable, $\mathbf{x}_{mt} \in \mathcal{X}$, which follows the transition probability $f(\mathbf{x}_{m,t+1}|\mathbf{a}_{mt}, \mathbf{x}_{mt}) = \begin{cases} 1 & \text{if } \mathbf{x}_{m,t+1} = \mathbf{a}_{mt} \\ 0 & \text{otherwise} \end{cases}$.

Firms observe the lagged pricing $\mathbf{x}_{mt}$, and a state variable, $h_t \in \mathcal{H}$, which is common to all drug brands and does not impact payoffs. The vector of private

[12] In this data set, prices typically increase from low to high during the collusive period. However, the price choices of the model can be relaxed to account for multiple price levels or gradual price increases.

information shocks, ϵ_{imt}, is independent of $\mathbf{x}_{mt}$ and has a zero mean.

The one-period payoff function for firm i, market m at time t is given by:

$$\Pi_{imt} = \underbrace{\mathrm{R}_{im}(a_{imt}, \mathbf{a}_{-imt}, \mathbf{x}_{mt}; \boldsymbol{\theta}_{i1}) - \mathrm{A}_{im}(\mathbf{a}_{mt}, \mathbf{x}_{mt}; \boldsymbol{\theta}_{i2})}_{\equiv \pi_{im}(\mathbf{a}_{mt}, \mathbf{x}_{mt}, \boldsymbol{\theta}_i)} + \epsilon_{imt}(a_{imt}),$$

Here, firm i's expected flow payoff from market m is governed by parameter $\boldsymbol{\theta}_i = (\boldsymbol{\theta}_{i1}, \boldsymbol{\theta}_{i2})$. The terms R_{im} and A_{im} represent variable profit and adjustment cost, respectively, and are determined by the parameters $\boldsymbol{\theta}_{i1}$ and $\boldsymbol{\theta}_{i2}$ for each firm i. I assume that the error terms, $\epsilon_{imt}(a_{imt})$, are independent and identically distributed Type I extreme value random variables across firms and over time.

Variable Profit

The variable profit is calculated from a basic logit demand model, premised on the assumption of homogeneous consumer behaviour. In each market, a consumer can purchase from any of the three available stores. Consider a consumer buying product m from firm i at time t. The logit demand model suggests the following closed-form representation of the market share:

$$\ln(s_{imt}) - \ln(s_{0mt}) = b_m - \alpha_m p_{imt} + \xi_{im}^{(1)} + \xi_{imt}^{(2)} \tag{1}$$

Here, s_{0mt} represents the share of the outside product in market m. The quantity is calculated using $q_{imt}(\mathbf{a}_m) = z_m^{\text{Size}} s_{imt}(\mathbf{p}_{mt})$, where $\mathbf{p}_{mt}$ is a vector of all firms' prices and z_m^{Size} the average total sales volume for market m across all three chains devided by 0.9. Based on the expert report, roughly 90% of sales occur through the three chain stores, with the remaining 10% of consumers purchasing from smaller local stores.

In the demand estimation, I apply Ordinary Least Squares (OLS) regression to the data from January 2006 to November 2006, a period characterized by competition. During this time, the primary source of price variation stemmed from weekly discounts offered by the firms. These firms would enact a price cut, often exceeding a 15% decrease, on a certain day, only to revert to the original price the following day. While potential endogeneity concerns may arise with the use of OLS, I argue that, during this period, the substantial fluctuations in price levels cannot be attributed solely to price endogeneity caused by demand shock. [13]

In this study, I have obtained a preliminary measure of wholesale prices for Salcobrand, spanning from November 1st, 2007, to May 1st, 2008. This data set includes all the products for which there are allegations of collusion among drug stores. I utilize the mean of these wholesale prices to represent the marginal cost of the products, denoted as c_{imt}. Expert reports suggest that the three pharmacy chains under consideration have comparable negotiating power with the distributor. Therefore, it is a reasonable assumption that these chains face similar marginal costs. The expected profit, given the pricing decisions, is represented as the flow payoff:

$$\mathrm{R}_{im}(\mathbf{a}_{mt};\boldsymbol{\theta}_{i1}) \equiv \mathbb{E}[(p_{imt}(a_{imt}) - c_{imt})z_m^{\mathrm{Size}} s_{imt}(\mathbf{a}_{mt})] \tag{2}$$

where $p_{im}(0)$ and $p_{im}(1)$ represent the median prices before and after the coordinated price increase; z_m^{Size} represents the market size, computed as the average weekly volume for product m; and $s_{imt}(\cdot)$ and c_{imt} are market shares and marginal costs derived from estimated demand functions respectively. [14]

Adjustment Cost

The adjustment cost, denoted as $\mathrm{A}_{im}(\mathbf{a}_{mt}, \mathbf{x}_{mt}; \boldsymbol{\theta}_{i2})$, is a critical element in the overall cost structure. This cost comprises three components as follows:

$$\mathrm{A}_{im}(\mathbf{a}_{mt}, \mathbf{x}_{mt}; \boldsymbol{\theta}_{i2}) = \mathrm{MC}_{im} * \big(\mathbb{1}(a_{imt} \neq a_{imt-1})\big) + \mathrm{LC}_{im} * \big(a_{imt}\mathbb{1}(\mathbf{a}_{-imt} = \mathbf{0})\big), \tag{4}$$

Here, MC_{im} refers to the market-level menu cost, and LC_{im} represents the lead-

[13] In addition to the OLS approach, I also explored the application of Instrumental Variables (IV) regression and random-coefficient logit estimation. The results from these alternative methodologies did not significantly affect the structural parameter of interest in the demand specification, thereby reinforcing the robustness of my findings.

[14] In an alternative analysis, I leverage the demand estimation to infer the marginal cost for the estimation, employing daily observations of the price p_{imt} and market share s_{imt} for product m from firm i at time t. With firms striving to maximize their daily profit, I estimtae the following regression to obtain the marginal costs:

$$p_{imt} = \arg\max_p (p - c_{imt})s_{imt} \tag{3}$$

This exercise does not significantly affect my structural parameter estimation results, thereby underscoring the validity of using OLS for estimation.

ership cost. The market-level menu costs, MC_{im}, are associated with price adjustments. These costs cover the one-time labour expenses required to update price lists and communicate these changes to customers and stakeholders.

Leadership costs, LC_{im}, are incurred when firm i is the first the increase the price on market m. Leadership costs refer to the potential losses associated with being a price leader. For instance, there is a risk of losing customers to competitors. Consumers often buy multiple products during a single visit, so a price increase for one item may prompt them to switch to other stores for their remaining purchases. This amplifies the overall costs of leadership.

Assuming uniform menu costs across markets, leadership costs are modelled as functions of market sizes and profit differences.

$$\mathrm{MC}_{im} = \gamma_i^{\mathrm{MC},0},\ \mathrm{LC}_{im} \quad = \gamma_i^{\mathrm{LC},0} + \gamma_i^{\mathrm{LC,Profit}} z_{im}^{\mathrm{Profit}} + \gamma_i^{\mathrm{LC,Size}} \log(z_m^{\mathrm{Size}}), \tag{5}$$

Here, z_{im}^{Profit} is the expected profit difference between colluding and remaining in competition, computed from the estimated demand. Further, $\log(z_m^{\mathrm{Size}})$ is the logarithm of the market size for m. The adjustment cost is summarized by the parameter vector $\boldsymbol{\theta}_{i2} = [\gamma_i^{\mathrm{MC},0}, \gamma_i^{\mathrm{LC},0}, \gamma_i^{\mathrm{LC,Profit}}, \gamma_i^{\mathrm{LC,Size}}]$, which includes the coefficients for menu cost and leadership cost.

3.3 Policy Functions

By leveraging the characteristic of h_t being non-payoff-relevant, I allow it to serve as an exclusion restriction. The assumptions of the model are summarized below.

Assumption 1. *For any market m, I assume the following conditions hold:*

(A) Firms' policy functions are dependent on payoff-relevant state variables, denoted by $\mathbf{x}_{mt}$ and ϵ_{imt}, as well as a public non-payoff-relevant variable, represented by h_t. Firms' beliefs regarding other firms' strategies on market m are contingent on $(\mathbf{x}_{mt}, h_t)$.

(B) Firms are forward-looking, maximizing inter-temporal payoffs given their beliefs.

(C) A firm's beliefs about its own future actions are unbiased expectations of actual future actions.

(D) Firms' private information ϵ_{imt} is independently distributed across firms and follows the Type I extreme value distribution G_ϵ, where G_ϵ is common knowledge.

I begin by discussing the implications of Assumption 1 (A), which posits that the strategy function for firm i at period t is denoted as $\sigma_{imt}(\mathbf{x}_{mt}, h_t, \epsilon_{imt})$. This assumption deviates from the standard MPE framework, which assumes firms' policy functions are contingent on payoff-relevant state variables $(\mathbf{x}_{mt}, \boldsymbol{\epsilon}_{imt})$. Instead, my assumption posits that firms' policy functions are dependent on a public non-payoff-relevant variable h_t. [15] Assumption 1 (B) asserts that firms are rational and their policy functions represent the best response given inter-temporal payoffs and beliefs about other firms' actions. This assumption is a standard feature of dynamic games and is crucial for analyzing strategic interactions. Assumption 1 (C) posits that firms have unbiased beliefs about their own behaviour, implying they comprehend their responses given future states, beliefs, and observed payoff-relevant variables. This assumption is crucial for firms to make optimal decisions based on their expectations of future outcomes. Lastly, Assumption 1 (D) requires that firms' beliefs meet the condition that other firms' actions are independent of each other, conditional on common knowledge state variables. This simplifies computation and reduces the number of free beliefs, making the model easier to analyze. Assumption 1 enables a more flexible and realistic approach to model firm behaviour. Given a strategy function σ_{imt}, I define the Conditional Choice Probability (CCP) representation, denoted as $\mathbf{P}_{im}(a, \mathbf{x}, h)$, to describe the likelihood of firm i selecting action a under the conditions $\mathbf{x}_{mt} = \mathbf{x}$ and $h_t = h$.[16]

In contrast to the standard MPE framework, I relax the assumption that firms' beliefs about other firms' actions are in alignment with their actual actions. Instead, I introduce a probability function, $\mathbf{B}_{im}(\mathbf{a}_{-im}, \mathbf{x}_{mt}, h_t)$, which represents firm i's beliefs about the actions of other firms, given the history h_t and common knowledge state variables.[17] This model includes the standard MPE in a particular case

[15] It's important to note that the variable h_t can be adapted to various contexts. In the current application, h_t represents the number of colluded markets at time t. In contrast, in a different study like the one by Byrne and De Roos (2019), h_t could represent the number of observed price jumps on a particular weekday if the goal is to comprehend how dominant firms experiment to establish price leadership.

[16] The CCP functions are useful for characterizing firm behaviour. When state variables $\mathbf{x}_{mt}$ and h_t have discrete support, the CCP function can be expressed as a finite-dimensional vector $\mathbf{P}_{im}$. In this article, the term $\mathrm{P}_{im}(\cdot)$ or the vector $\mathbf{P}_{im}$ is employed to represent the actual CCP of firm i at period t.

[17] For state variables with finite support, the belief function for a given h_t is represented as $\mathbf{B}_{im}(\mathbf{a}_{-im}, \mathbf{x}_{mt}, h_t)$. This function encompasses the probabilities of various actions by other firms, denoted as $\mathrm{B}_{im}(\mathbf{a}_{-im}, \mathbf{x}_{mt}, h_t)$, and is subject to the constraint that the sum of these probabilities is

when the equilibrium belief assumption is satisfied, which can be expressed as $\mathbf{B}_{im} = \Pi_{j \neq i} \mathbf{P}_{jm}$ for every firm i at every history h_t.

This framework differs from that of Aguirregabiria and Magesan (2020) in a non-trivial way: I incorporate h_t, a conditioning variable that, while not directly affecting any firm's payoff, is nonetheless present in the policy function. This distinction is essential, as including such a variable can provide valuable insights into how agents coordinate and choose among multiple equilibria.

Given the firm i's beliefs, $\mathbf{B}_{im}$, firm i's *policy function* is the optimal solution to a single-agent dynamic programming problem. The dynamic programming problem can be described in terms of (1) a discount factor β; (2) a sequence of expected one-period payoff functions $\{\pi_i^{\mathbf{B}_{im}}(\mathbf{a}_{im,t+s}, \mathbf{x}_{m,t+s}, h_{t+s}) + \epsilon_{im,t+s}(a_{im,t+s})\}_{s=0,1,\ldots,T-t}$, where $\pi_i^{\mathbf{B}_{im}}(\mathbf{a}_{mt}, \mathbf{x}_{mt}, h_t) = \sum_{a_{-imt} \in \mathcal{A}_{-i}} \pi_i(\mathbf{a}_{imt}, \mathbf{a}_{-imt}, \mathbf{x}_{mt}) \mathbf{B}_{im}(\mathbf{a}_{-im}, \mathbf{x}_{mt}, h_t)$; and (3) a sequence of transition probability functions $\{f_i^{\mathbf{B}_{im}}(\tilde{\mathbf{x}}|\mathbf{a}_{im}, \mathbf{x}_{t+s}), f_{h,i}(\tilde{h}|h_{t+s})\}_{s=0,1,\ldots,T-t}\}$, where $f_i^{\mathbf{B}_{im}}(\tilde{\mathbf{x}}|\mathbf{a}_{mt}, \mathbf{x}_{mt}, h_t) = \sum_{a_{-imt} \in \mathcal{A}_{-i}} f(\tilde{\mathbf{x}}|\mathbf{a}_{imt}, \mathbf{a}_{-imt}, \mathbf{x}_{mt}) \mathbf{B}_{im}(\mathbf{a}_{-imt}, \mathbf{x}_{mt}, h_t)$. [18] Let $\mathrm{V}_{im}^{\mathbf{B}_{im}}(\mathbf{x}_{mt}, h_t, \epsilon_{imt})$ be the value function for firm i's dynamic programming problem given his beliefs, by Bellman's principle, the sequence of value functions can be obtained recursively using the following Bellman equation:

$$\mathrm{V}_{im}^{\mathbf{B}_{im}}(\mathbf{x}_{mt}, h_t, \epsilon_{imt}) = \max_{a_{im}} \left\{ v_{im}^{\mathbf{B}_{im}}(a_{im}, \mathbf{x}_{mt}, h_t) + \epsilon_{imt}(a_{im}) \right\},$$

where $v_{im}^{\mathbf{B}_{im}}(\mathbf{x}_{mt}, a_i)$ is the conditional choice value function

$$\begin{aligned} &\mathrm{v}_{im}^{\mathbf{B}_{im}}(a_{imt}, \mathbf{x}_{mt}, h_t) = \pi_{im}^{\mathbf{B}_{im}}(a_{imt}, \mathbf{x}_{mt}, h_t) \\ &+ \beta \sum_{\mathbf{x}_{t+1} \in \mathcal{X}} f_i^{\mathbf{B}_{im}}(x_{t+1}|\mathbf{a}_{mt}, \mathbf{x}_{mt}) \sum_{h_{t+1}} f_{h,i}(h_{t+1}|h_t) \int \mathrm{V}_{im}^{\mathbf{B}_{im}}(x_{m,t+1}, h_{t+1}, \epsilon_{im,t+1}) dG_{it}(\epsilon_{it+1}). \end{aligned} \tag{6}$$

I obtain the CCP representation of the best response function integrating the policy function over the distribution of ϵ_{imt},

$$\begin{aligned} \mathbf{P}_{im}(\mathbf{a}, \mathbf{x}_{mt}, h_t) &= \int \left\{ \boldsymbol{\epsilon}_{im}(\mathbf{a}') - \boldsymbol{\epsilon}_{im}(\mathbf{a}) \leq v_{im}^{\mathbf{B}_{im}}(\mathbf{a}, \mathbf{x}_{mt}, h_t) - v_{im}^{\mathbf{B}_{im}}(\mathbf{a}', \mathbf{x}_{mt}, h_t) \text{ for any } \mathbf{a}' \neq \mathbf{a} \right\} dG_{it}(\boldsymbol{\epsilon}_{imt}) \\ &= \Lambda_{im}\Big(\mathbf{a}; \tilde{\mathbf{v}}_{im}^{\mathbf{B}_{im}}(\mathbf{x}_t, h_t)\Big), \end{aligned} \tag{7}$$

equal to 1. The belief function $\mathbf{B}_{im}$ belongs to the set $\mathcal{B}$, which is a subset of $[0,1]^{A^{\mathcal{I}-1}|\mathcal{X}||\mathcal{H}|}$.

[18] Regarding the evolution of history, I believe that a single market's contribution does not impact the overall evolution of the history variable. However, a firm may have a different perspective on how the history variable may evolve compared to other firms.

where $\Lambda(\mathbf{a};\cdot)$ is the C.D.F. of the vector of $\{\boldsymbol{\epsilon}_{im}(\mathbf{a}) - \boldsymbol{\epsilon}_{im}(\mathbf{0}) : \mathbf{a} \in \mathcal{A} \setminus \{\mathbf{0}\}\}$and $\tilde{\mathbf{v}}_{im}^{\mathbf{B}_{im}}(\mathbf{x}_{mt}, h_t)$ is the vector of the continuation value differences.

3.4 Firms' Incentives - A simplified Model

In this section, I include a simple model to explain how "trust" may affect firms' incentive to lead and follow. Consider two categories of prescription drugs (represented as markets), denoted $m = 1, 2$, and two firms, indicated as $i = 1, 2$. At a specific time t, these firms will observe the actions from the previous period $t-1$, symbolized as $\mathbf{x}_{mt} = \mathbf{a}_{t-1}$, and determine their next course of action. Here, $a_{imt} = 1$ signifies a collusive price, while $a_{imt} = 0$ signals a competitive price.

The decision that a firm makes hinges on the preceding actions. For example, if $\mathbf{x}_{mt} = \mathbf{x}_m^{\text{Compete}} = (0,0)$, the firm must resolve whether to prolong the waiting period. If $\mathbf{x}_{mt} = \mathbf{x}_m^{\text{Lead}} = (1,0)$ and $a_{imt-1} = 1$, the firm must decide if it should continue to retain its leadership position. If $\mathbf{x}_{mt} = \mathbf{x}_m^{\text{Follow}} = (0,1)$ and $a_{imt-1} = 0$, the firm must determine whether to accompany a price increase. Finally, if $\mathbf{x}_{mt} = \mathbf{x}_m^{\text{Collude}} = (1,1)$, the firms have already escalated prices within this market to the collusive level, a state considered as a terminal state. We hypothesize that the profit derived from selling the product is denoted by $\mathrm{R}_{im}(\mathbf{x}_{mb})$. Intuitively, order these profits as follows: $\mathrm{R}_{im}(\mathbf{x}_m^{\text{Lead}}) < \mathrm{R}_{im}(\mathbf{x}_m^{\text{Compete}}) < \mathrm{R}_{im}(\mathbf{x}_m^{\text{Collude}}) < \mathrm{R}_{im}(\mathbf{x}_m^{\text{Follow}})$.

Leader's Problem with Trust-Building

To simplify the discussion, let's represent $\mathrm{V}(\mathbf{x}_m^{\text{Collude}})$ as $\frac{1}{1-\beta}\mathrm{R}_{im}(\mathbf{x}_m^{\text{Collude}})$. This notation assumes that the companies will uphold their collusive agreement indefinitely in this market context. Furthermore, firm i, as the determined price leader, has a single opportunity to lead. This assumption implies that if the price increase initiated by the leader does not succeed, all firms are destined to remain in perpetual competition. In the role of the price leader, firm i anticipates that the following firm i''s likelihood of adhering is represented as $\phi_m^{(1)}(h) = \mathrm{B}_i(1, \mathbf{x}^{\text{Lead}}, h)$. This representation illustrates firm i's belief about firm i''s action, but it does not necessarily align with the actual probability of that action. Moreover, $\partial\phi_m^{(1)}(h)/\partial h > 0$, the belief increases with number of succesful coordination.

When the state is $\mathbf{x}^{\text{Compete}}$, a firm's decision to set a higher price essentially represents its choice to assume the role of a price leader. The choice-dependent value function for player i, determining whether to initiate a price increase, can be expressed as follows:

$$\begin{aligned}
\mathrm{v}_{im}(a_{imt}=1,\mathbf{x}^{\text{Compete}},h) &= \mathrm{R}_{im}(\mathbf{x}_m^{\text{Lead}}) - \big(\mathrm{MC}_{im}+\mathrm{LC}_{im}\big) + \beta\Big(\phi_m^{(1)}(h)\frac{\mathrm{R}_{im}(\mathbf{x}_m^{\text{Collude}})}{1-\beta} + \beta(1-\phi_m^{(1)}(h))\frac{\mathrm{R}_{im}(\mathbf{x}_m^{\text{Compete}})}{1-\beta} \\
&\quad + (1-\phi_m^{(1)}(h))\mathrm{R}_{im}(\mathbf{x}_m^{\text{Lead}})\Big) \\
\mathrm{v}_{im}(a_{imt}=0,\mathbf{x}^{\text{Compete}},h) &= \frac{\mathrm{R}_{im}(\mathbf{x}_m^{\text{Compete}})}{1-\beta}.
\end{aligned}$$

Then I have the incentive to lead is determined by $\mathrm{p}_{im}^{\text{Lead}} = \mathrm{P}_{im}(a_{imt}=1,\mathbf{x}^{\text{Compete}},h) = \Lambda(\tilde{\mathrm{v}}_{im}(a_{imt}=1,\mathbf{x}^{\text{Compete}},h))$ and $\tilde{\mathrm{v}}_i(a_{imt}=1,\mathbf{x}^{\text{Compete}},h) = \mathrm{v}_{im}(a_{imt}=1,\mathbf{x}^{\text{Compete}},h) - \mathrm{v}_{im}(a_{imt}=0,\mathbf{x}^{\text{Compete}},h)$.

$$\begin{aligned}
\tilde{\mathrm{v}}_{im}(a_{imt}=1,\mathbf{x}^{\text{Compete}},h) &= \underbrace{\big(1+\beta(1-\phi_m^{(1)})\big)\mathrm{R}_{im}(\mathbf{x}_m^{\text{Lead}}) - \Big(\mathrm{MC}_{im}+\mathrm{LC}_{im}\Big)}_{\text{Flow payoff during the period of leading}} \\
&\quad \underbrace{+\beta\Big(\frac{\phi_m^{(1)}}{1-\beta}\underbrace{\big(\mathrm{R}_{im}(\mathbf{x}_m^{\text{Collude}}) - \mathrm{R}_{im}(\mathbf{x}_m^{\text{Compete}})\big)}_{\text{Profit Difference } z_{im}^{\text{Profit}}}\Big) - \big(1-\phi_m^{(1)} - \beta(1-\phi_m^{(1)})\big)\frac{\mathrm{R}_{im}(\mathbf{x}_m^{\text{Compete}})}{1-\beta}}_{\text{Future payoff difference if succesful collude}}
\end{aligned} \tag{8}$$

As the trust build up, $\phi_m^{(1)}$ increases. Overall, the leader's incentive to lead increases as trust builds up; as $\phi_m^{(1)}(h)$ increases with h, the probability of future payoff gets larger, and the possible loss from failing decreases. Suppose that for market $m=1$, if the profit difference is larger than that of $m=2$, $\pi_{i1}(\mathbf{x}_1^{\text{Collude}}) - \pi_{i1}(\mathbf{x}_1^{\text{Compete}}) > \pi_{i2}(\mathbf{x}_2^{\text{Collude}}) - \pi_{i2}(\mathbf{x}_2^{\text{Compete}})$, then $\mathrm{p}_{i1}^{\text{Lead}} > \mathrm{p}_{i2}^{\text{Lead}}$, firms are more inclined to lead on market 1.

Follower's Problem with Trust-Building

For the follower's problem, assume i' is now the price leader and i will take the follower position. The leader waits at most three days before taking the lead, denoted by $\tau = 0,1,2$. The leader will not increase the price on the first day (when $\tau=0$). On the second day (when $\tau=1$), the follower will assume that the leader's probability of waiting is $\phi_{im}^{(2)}(h) = \mathrm{B}_i(1,\mathbf{x}^{\text{Follow}},h)$, which increases with h. When $\tau=2$, the leader will revert the price increase on the third day. Therefore, it is optimal for the follower to follow at $\tau=2$ if not followed on $\tau=0$. To simplify the scenario, assume that the follower will follow at $\tau=2$. Furthermore, if the follower waits at $\tau=0$, they will only increase the price at $\tau=2$.

When the state is $\mathbf{x}^{\text{Follow}}$, a firm's decision to set a higher price represents its choice to follow the price increase. The choice-dependent value function for player i, determining whether to follow on the first day observing the price leader has moved, can be expressed as follows:

$$
\begin{aligned}
\mathrm{v}_{im}(a_{imt}=1,\mathbf{x}^{\text{Compete}},h) &= \frac{\pi_{im}(\mathbf{x}_m^{\text{Collude}})}{1-\beta} - \mathrm{MC}_{im}, \\
\mathrm{v}_{im}(a_{imt}=0,\mathbf{x}^{\text{Compete}},h) &= \pi_{im}(\mathbf{x}_m^{\text{Follow}}) + \beta\Big(\big(1-\phi_m^{(2)}(h)\big)\frac{\pi_{im}(\mathbf{x}_m^{\text{Compete}})}{1-\beta} + \phi_m^{(2)}(h)\big(\pi_{im}(\mathbf{x}_m^{\text{Follow}}) \\
&\qquad + \beta\big(\frac{\pi_{im}(\mathbf{x}_m^{\text{Collude}})}{1-\beta} - \mathrm{MC}_{im}\big)\big)\Big).
\end{aligned}
$$

Then I have the incentive to follow in the first day observing the state is determined by $\mathrm{p}_{im}^{\text{Follow}} = \mathrm{P}_{im}(a_{imt}=1,\mathbf{x}^{\text{Follow}},h) = \Lambda(\tilde{\mathrm{v}}_{im}(a_{imt}=1,\mathbf{x}^{\text{Follow}},h))$ and $\tilde{\mathrm{v}}_i(a_{imt}=1,\mathbf{x}^{\text{Follow}},h) = \mathrm{v}_{im}(a_{imt}=1,\mathbf{x}^{\text{Follow}},h) - \mathrm{v}_{im}(a_{imt}=0,\mathbf{x}^{\text{Follow}},h)$.

$$
\begin{aligned}
\tilde{\mathrm{v}}_{im}(a_{imt}=1,\mathbf{x}^{\text{Follow}},h) = {} & \frac{(1-\beta^2\phi_m^{(2)}(h))}{1-\beta}\underbrace{\big(\pi_{im}(\mathbf{x}_m^{\text{Collude}}) - \pi_{im}(\mathbf{x}_m^{\text{Compete}})\big)}_{\text{Profit Difference } z_{im}^{\text{Profit}}} - (1-\beta^2\phi_m^{(2)}(h))\mathrm{MC}_{im} \\
& - (1+\beta\phi_m^{(2)}(h))\pi_{im}(\mathbf{x}_m^{\text{Follow}}) - (\beta - 1 - \beta\phi_m^{(2)}(h) + \beta^2\phi_m^{(2)}(h))\frac{\pi_{im}(\mathbf{x}_m^{\text{Compete}})}{1-\beta}
\end{aligned}
\tag{9}
$$

As the value of $\phi_m^{(2)}(h)$ increases, the follower becomes more convinced that the leader is willing to wait. The belief change reduces the follower's motivation to increase the price earlier. As a result, the time before a price increase is followed is extended, leading to a rise in the number of days that the follower waits before following.

3.5 Equilibrium Restriction

Within the MPE framework, it is common to impose the equilibrium belief assumption, stating that a firm's beliefs about other firms' behaviours are unbiased expectations of their actual behaviour. In this subsection, I introduce the concept of equilibrium, referred to as TBE, and the associated equilibrium restriction. I define the biased belief function as a function mapping the "history" variable to a value between 0 and 1: $\lambda : \mathcal{H} \to [0,1]$. Moreover, I impose the following assumption on how firms' beliefs are determined:

Assumption 2 (Belief formation). *For all $h \in \mathcal{H}$, the belief of the game $\mathbf{B}^h$ is written as*

$$\mathbf{B}_i(\cdot, \mathbf{x}, h) = \Xi(\lambda(h), \mathbf{P}_{-i}(\cdot, \mathbf{x}, h)), \tag{10}$$

where $\mathbf{P}_{-i}(\cdot, \mathbf{x}, h) = \Big\{\Pi_{i' \in \mathcal{I}, i' \neq i} \mathbf{P}_{i'}(a_{i'}, \mathbf{x}, h), a_{i'} \in \mathcal{A}\Big\}$ is a vector of other firms' choice probabilities given the state $\mathbf{x}$, $\mathbf{B}_i(\cdot, \mathbf{x}, h) = \Big\{\mathbf{B}_i(\mathbf{a}_{-it}, \mathbf{x}, h) : \mathbf{a_{-it}} = \{a_{i'} \in \mathcal{A} : i' \in \mathcal{I}, i' \neq i\}\Big\}$ is firm i's beliefs given state $\mathbf{x}$. The function Ξ is a separable mapping that takes the arguement of $\lambda(\cdot)$ and $\mathbf{P}$, which can be written as $\Xi(\lambda(h), \mathbf{P}_{-i}(\cdot, \mathbf{x}, h)) = \mathbf{M}^{\lambda(h)}\mathbf{P}$, where $\mathbf{M}^{\lambda(h)}$ is an injective function of only $\lambda(h)$. ■

In this application, I select Ξ-mapping and the associated $\mathbf{M}^{\lambda(h)}$ as the following: when $h_t = h$, the belief of firm i is

$$\mathbf{B}_i(a_{-it}, \mathbf{x}_t, h_t) = \Pi_{i' \neq i}\Big(a_{i't}\lambda(h_t)\mathbf{P}_{i'}(a_{i't}, \mathbf{x}_t, h_t) + (1 - a_{i't})(1 - \lambda(h_t))\mathbf{P}_{i'}(a_{i't}, \mathbf{x}_t, h_t)\Big), \tag{11}$$

where $\lambda(h_t) \in [0, 1]$ is a function of history. This equation represents a firm-specific bias between firms' beliefs and other firms' "true" CCPs. This specification allows firms to express uncertainty regarding other firms' mindsets between two potential strategies: competitive and collusive price leadership. One possible interpretation is that lambda represents the Bayesian posterior probability across these strategies, updating based on observed history.

Formally, write $\mathbf{P} \equiv \{\mathbf{P}_i(a_i, \mathbf{x}, h), a_i \in \mathcal{A}, \mathbf{x} \in \mathcal{X}\}_{i \in \mathcal{I}, h \in \mathcal{H}}$ as the fixed point to the mapping of

$$\mathbf{P} = \Psi_{\boldsymbol{\lambda}}(\mathbf{P}), \tag{12}$$

where $\Psi_{\boldsymbol{\lambda}}$ is defined by the mapping such that $\mathbf{P}_i(a_i, \mathbf{x}, h) = \Lambda_{im}\Big(a_i; \tilde{\mathbf{v}}_i^{\mathbf{B}_{im}}(\mathbf{x}, h)\Big)$ and $\mathbf{B}_i(\cdot, \mathbf{x}, h) = \Xi(\lambda(h), \mathbf{P}_{-i}(\cdot, \mathbf{x}, h))$, where the best response mapping Λ_{it} is as defined by (7). The case $\boldsymbol{\lambda}^{\text{MPE}} = \{\lambda(h) = 1 \text{ for } h \in \mathcal{H}\}$ corresponds to the solution of an MPE.

4 Identification and Estimation

This section discusses the identification of the structural parameters $\{\boldsymbol{\theta_i}\}_{i \in \mathcal{I}}, \lambda(\cdot)$. I assume the data are generated from a TBE discussed in section 3.5. Suppose the researcher observes panel data $\{\mathbf{a}_{mt}, \mathbf{x}_{mt}\}$ over periods $t \in \{1, 2, \ldots, T_{data}\}$ and markets $m \in \mathcal{M}$. I assume the payoff on the market m depends only on $\mathbf{x}_{mt}$. Let $\mathbf{P}$ be the vector of the CCPs of the true conditional probabilities in the population and let $\hat{\mathbf{P}}$ denote the CCPs in the sample. I use the sample to estimate the flow payoff function π_{im} and the biased belief functions $\{\lambda(h)\}_{h\in\mathcal{H}}$ (i.e., the structural parameters), given the transition densities of the state variable $f(\mathbf{x}_{t+1}|\mathbf{a}_t, \mathbf{x}_t)$ and $f_h(h_{t+1}|\mathbf{a}_t, h_t)$, the CCP mapping $\Lambda_{it}(\cdot)$, and the discount factor β.

4.1 Identification of Payoff Parameters

The payoff functions in this model are comprised of two components: the market-level variable profit function denoted as R_{im}, and the adjustment cost function denoted as A_{im}. The profit function R_{im} is estimated using the demand function and the measure for marginal costs. The adjustment costs A_{im} are identified based on the revealed preference of the decisions made during the coordination period, which took place from December 2007 to April 2008.

Estimating adjustment costs relies on the revealed preference, assuming firms compete in an infinitely repeated dynamic price-setting game in each market. I now show the structural payoff parameters $\boldsymbol{\theta}_{i2}$ for the adjustment cost functions can be identified, assuming the biased belief functions $\lambda(\cdot)$, the discount factor β, and the transition density, $f(\mathbf{x}_{t+1}|\mathbf{a}_t, \mathbf{x}_t)$ and $f_h(h_{t+1}|\mathbf{a}_t, h_t)$, are known.

Assumption 3 (Best Response)**.** *For every firm i, $\mathbf{P}_i$ is the best response in period t given the firm's beliefs $\mathbf{B}_i$ and the payoff function.* ■

Assumption 3 is critical for identifying the value differences. This assumption implies that firms are rational in that their actual behaviour is the best response, given their beliefs about their own and others' actions.

To identify belief and structural parameters, I adapt exclusion restrictions from Aguirregabiria and Magesan (2020) (Assumption ID-3) and impose Assumption 4.

Assumption 4 (Exclusion restriction)**.** *In market m, the payoff depends on subvector $\mathbf{x}_{mt}$ satisfy: (A) Lagged price x_{imt} of firm i in market m at time t only affects firm i's payoff function for market m but not other markets or firms. (B) The transition probability of the state variable $\mathbf{x}_{mt}$ is separable from the state variables on other markets. The state variable*

x_{imt} is such that the value of x_{imt+1} does not depend on (x_{imt}) once I condition on a_{imt}. (C) The joint distribution of $(x_{imt}, \mathbf{x}_{-imt})$ over the population of M markets where I observe these variables has a strictly positive probability at every point in the joint support set $\mathcal{X}$. (D) The flow payoff function $\boldsymbol{\pi}_{im}(a_{imt}, a_{-imt}, x_{imt})$ is invariant across history $h \in \mathcal{H}$. (E) The transition of the history does not depend on the payoff-relevant state variable $\mathbf{x}_t$: $f_h(h_{t+1}|\mathbf{a}_t, h_t)$. ■

The exclusion restrictions are critical in identifying the biased belief function, denoted as $\lambda(h)$. Assumption 4(A) stems directly from the definition of adjustment cost: the prior pricing decision of firm i impacts only its own adjustment cost. This leads to the formulation of the flow payoff function as $\pi_{im}(a_{imt}, a_{-imt}, x_{imt})$. Assumption 4(B) implies that a pricing decision made two periods prior does not influence the transition of the subsequent state variable. Assumption 4(C) requires that the joint cross-sectional distribution of the state variables $(x_{imt}, \mathbf{x}_{-imt})$ has positive probability for all support values across the sample of M markets. Assumption 4(D) assumes that past interactions between firms do not influence current payoffs in any market. Lastly, Assumption 4(E) assumes that the evolution of market history is independent of the currently relevant market state.

Proposition 1. *Under assumptions 1, 2, 3, and 4, with known $\lambda(h)$ and $\mathbf{P}$, the structural payoff parameters $\boldsymbol{\theta}_{i2}$ are identified.*

4.2 Identification of Beliefs

This section discusses the identification of the biased belief functions $\lambda(h_t)$ as defined in section 3.5. The identification strategy follows that of Aguirregabiria and Magesan (2020), whereby the ratio of a function of beliefs $\mathrm{M}^{\lambda(h)}(\mathrm{M}^{\lambda(h')})^{-1}$ is identified for each pair of (h, h').

Proposition 2 (Identification of the ratio of beliefs)**.** *Under assumptions 1, 2, 3, and 4, the ratio of a function of the biased belief function is identified from $\mathrm{M}^{\lambda(h)}(\mathrm{M}^{\lambda(h')})^{-1}$ for each pair of histories $(h, h') \in \mathcal{H}$.*

Proposition 2 provides the identification of ratios of a function of the biased belief functions across any (h, h'). However, to identify the value of the biased belief function $\lambda(h)$ for any h, I need to know the value of $\lambda(h)$ for at least some $h \in \mathcal{H}$. I assume that the beliefs are unbiased for a subset of data, as follows:

Assumption 5 (Unbiased belief in the last period)**.** *For the last of the observed periods*

$$\bar{h} = \max(\mathcal{H}),$$

the firms' beliefs are unbiased everywhere: $\mathbf{B}_{im}(\boldsymbol{a}_{-i,m}, \mathbf{x}, \bar{h}) = \Pi_{i' \neq i} \mathbf{P}_{i'm}(a_{i'm}, \mathbf{x}, \bar{h})$*, where* $\boldsymbol{a}_{-im} = \{a_{i'm} : i' \neq i\}$ *for every firm* $i \in \mathcal{I}$*, every market* $m \in \mathcal{M}$*, and every possible state* $\mathbf{x}_m \in \mathcal{X}_m$*.* ■

With Assumption 5, $\mathrm{M}^{\lambda(h)}$ is identified for all $h \in \mathcal{H}$. Assume that firms' beliefs are in equilibrium after successfully colluding on a large enough number of drugs, and $\lambda(\bar{h}) = 1$. Because the the value of $\mathrm{M}^{\lambda(h)}(\mathrm{M}^{\lambda(\bar{h})})^{-1}$ is identified for every h, with the above assumption, the function of $\lambda(h)$, $\mathrm{M}^{\lambda(h)}$, is identified for all histories $h \in \mathcal{H}$. The identification of $\lambda(h)$ follows as $\mathrm{M}^{\lambda(h)}$ is an injective function.

4.3 Estimation

In this subsection, I explore the algorithm for estimating structural parameters, where $\hat{\boldsymbol{\theta}}_{1i}$ denotes the estimated flow payoff parameters. To compute the variable profit functions $\mathrm{R}_{im}(\cdot, \boldsymbol{\theta}_1)$, I use the median price before the price war and after collusive price increases obtained from demand estimation.

To estimate $\hat{\boldsymbol{\theta}}_{2i}$, I partition the game history state h into four distinct grids, reducing the dimensionality of the estimation. Here, h represents the number of markets in which firms have successfully colluded to increase prices. I divide the history into the following four grids: $\{[0, 30], [31, 90], [90, 150], [150, \infty)\}$ [19]. To overcome bias in the first stage of non-parametric estimation and address the limitations of the two-step method, I employ the nested pseudo-likelihood (NPL) estimator proposed by Aguirregabiria and Mira (2007). In addition, I incorporate the equilibrium restriction specified by (10). As highlighted by Aguirregabiria and Mira (2007), the inclusion of equilibrium restrictions in the model estimation preserves identification power, even in the presence of multiple equilibria.

[19] The specification of the step function type of belief update is arbitrary. In the Supplementary Appendix, I present a continuously updated belief, implying that $\lambda(h)$ varies for each $h \in \mathcal{H}$. The results are robust when considering a continuously updated belief function: the flexible belief model yields a pattern of gradual price increases, whereas the equilibrium belief model does not produce such a pattern.

5 Results and Counterfactual

In this section, I report the estimation results of the TBE model, in which I account for firms' coordination during the initiation of cartel formation. I estimated a benchmark Markov Perfect Equilibrium model for comparison. The model enforces firms' equilibrium beliefs. In addition, the TBE and MPE models were used to evaluate counterfactual outcomes. The results show that the TBE framework provides more credible predictions that align closely with the data. This section proceeds as follows.

5.1 Estimation Results

I derive estimates for $\boldsymbol{\theta}_{i1} = [\{p_{im}(\cdot), z_m^{\text{Size}}, b_m, \alpha_m, \xi_{im}^{(1)}, c_m^0, \omega_{im}^{(1)}\}_{m \in \mathcal{M}}]$ using the demand system. The process involves employing an Ordinary Least Squares (OLS) estimate for the parameters in equation (1), utilizing data spanning from January 2006 to November 2007, a period before the initiation of collusion.

Table 3 presents the estimated belief parameters λ over histories and the estimated structural costs computed from the estimated θ's. Panel A displays the estimated λ, which increases with history, indicating that firms gain trust through observing successful coordination incidents. Panel B reports the estimated menu costs and leadership costs across markets under TBE and MPE respectively. According to the TBE model, estimated menu and leadership costs are similar across firms of similar scale. Since the three companies operate on a similar scale, it is sensible that their menu and leadership costs would be comparable. Additionally, the forecasted leadership costs for Salcobrand are lower than those predicted for Cruz Verde and FASA. This is consistent with the general trend of Salcobrand frequently taking on the role of the price leader.

Based on the MPE model, Salcobrand is estimated to have much lower leadership costs than Cruz Verde and FASA. This implies that if Salcobrand decides to increase its prices, it would incur significantly lower expenses. On the other hand, under the MPE framework, if Cruz Verde decides to increase the price of an average product, it would face a cost of 95.8 million Chilean pesos (approximately 191,600 USD in 2008) on average, which is unreasonable. Panel C displays the estimated structural parameters under both models. The presence of a positive parameter signals a rise in costs. The cost of leadership increases with differences in profit. This could be due to the loss of profit from other purchases as consumers switch to the followers. A negative intercept for leadership costs implies lower associated costs for smaller profit differences. The leadership costs for FASA and Salcobrand decrease with the market size, and the costs for Cruz Verde increase with the market size. However, the coefficient for the market size is relatively small. Meanwhile, the correlation between market size and profit difference is strong and crucial.

To demonstrate the accuracy of TBE versus MPE, I present the predictions via simulated paths of both models in Figure 3. As depicted in the figure, the MPE model cannot replicate the observed gradual price increase. It predicts that all collusions will occur instantaneously within three weeks. [20] In contrast, the TBE model, which accounts for firms' coordination, accurately captures the gradual development of collusion observed in the data. These model predictions emphasize the importance of considering firms' learning to coordinate and allowing their beliefs to be updated when modelling the initiation of collusion.

Table 4 presents the predictions of Salcobrands' probability of leading a price increase averaged across the markets for various histories. TBE predicts that firms' probability of leading the price increase over phases, while MPE indicates such probability does not vary too much: The MPE model overestimates the CCPs in the early periods (History 0-30, 30-90) and subsequently predicts the collusions happen instantly. Both TBE and MPE suggest a greater likelihood of Salcobrand leading a price increase throughout all phases than its two competitors. This implies that Salcobrand bears lower leadership costs. However, MPE's prediction for Salcobrand to lead is unrealistically high and can be interpreted as, if in an equilibrium where beliefs are true, and firms know the collusion profit, then Salcobrand will immediately lead the price increase.

5.2 Counterfactual Analysis

In this study, I scrutinize two forms of hypothetical policy interventions: (1) the introduction of price adjustment friction by multiplying the menu cost tenfold com-

[20]To examine the confidence interval of the model-predicted data, I generate in-sample predictions with a parametric bootstrapped 95% confidence interval in the Supplementary Appendix.

Table 3: Estimated Structural Parameters

Panel A: Estimation of Belief Parameters $\lambda(h)$		
History	Estimates.	Bootstrap.
History 0-30	0.1789	0.153 (0.089)
History 30-90	0.2930	0.439 (0.271)
History 90-150	0.5182	0.515 (0.170)
History 150+	1.0000	-

Panel B: Estimation of Structural Costs(1000 Chilean Pesos)						
	TBE			**MPE**		
Costs	*Cruz Verde*	*FASA*	*Salcobrand*	*Cruz Verde*	*FASA*	*Salcobrand*
Menu Cost	74.619	96.044	84.711	14.218	1334.256	107.641
Leadership Cost	1602.475	2238.598	1429.974	95804.489	323648.470	4.219
90% Quantile	3985.463	5265.212	3182.917	225968.155	727003.623	282.964
10% Quantile	79.059	164.346	59.532	10971.365	52475.795	227.776

Panel C: Estimation of Structural Costs Parameters						
		TBE			**MPE**	
Parameter	*Cruz Verde*	*FASA*	*Salcobrand*	*Cruz Verde*	*FASA*	*Salcobrand*
$\gamma^{\mathrm{MC}}, 0_i$	74.6188	96.0439	84.7112	14.2176	1334.2561	107.6409
	(22.8003)	(41.8131)	(38.0040)	(9155.2740)	(2250.9839)	(51.0140)
$\gamma^{\mathrm{LC}}, 0_i$	-213.2839	-242.7637	-218.2709	-5161.4021	-777.7465	-255.3117
	(62.9368)	(137.0481)	(72.3270)	(16489.2515)	(10772.5192)	(76.7080)
$\gamma^{\mathrm{LC}}, \mathrm{Size}_i$	0.0527	-0.5238	-0.4002	13.6036	-91.6494	-0.3254
	(0.8594)	(1.3888)	(0.7702)	(1189.9790)	(906.1534)	(0.4056)
$\gamma^{\mathrm{LC}}, \mathrm{Profit}_i$	8.9545	10.7311	9.1895	486.7926	1423.1862	1.6946
	(1.3819)	(4.2826)	(0.2825)	(1516.7731)	(1864.8742)	(0.8675)

[1] Panel A presents the model estimations for $\hat{\lambda}_i(h)$. The standard deviation, computed from 499 parametric bootstrap replications, is reported within brackets in Panel A. This bootstrap procedure involves drawing data from M markets with replacement and performing the estimation accordingly.
[2] In Panel B, I calculate each drug's projected menu and leadership costs. The 10% and 90% quantiles of the computed costs are displayed in brackets across markets, representing the distribution of leadership costs across these markets. These costs are calculated based on the adjustment cost specifications outlined in Equation (5).
[3] In panel C, I report the estimated structural parameters as specified in section In the brackets, I report the bootstrapped standard deviation of the estimated parameters.

pared to the estimated values, and (2) the implementation of a structural remedy, such as a divestiture, which would require each chain to sell 25% of its stores to a new competitor. These interventions are suggested by Harrington (2018). An increase in menu costs influences firms' collusion incentives via both payoff and coordination effects. As the cost of adjusting prices escalates, the price leader may

Figure 3: Model Prediction - Simulated Path

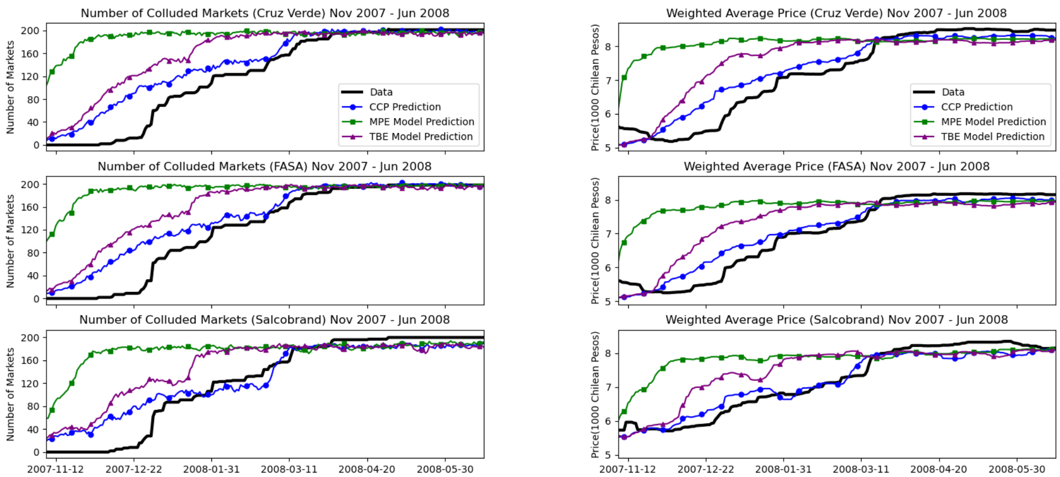


Table 4: Prediction of Firms' Price Leadership Probabilities

	Cruz Verde		*FASA*		*Salcobrand*	
	TBE	*MPE*	*TBE*	*MPE*	*TBE*	*MPE*
[0-30]	0.0104	0.0069	0.0155	0.0000	0.0316	0.6096
[30-90]	0.0451	0.0058	0.0846	0.0000	0.0375	0.5887
[90-150]	0.0484	0.0061	0.0104	0.0000	0.1101	0.6092
[150+]	0.2898	0.0058	0.2558	0.0000	0.4725	0.6742
All	0.0984	0.0062	0.0916	0.0000	0.1629	0.6204

[1] The table reports the mean probability of Salcobrand initiating a price increase affecting 204 products. [2] Under MPE, the probability of FASA leading a price increase essentially hit 0 because I bind the probability of leading to be 1e-6.

become hesitant to elevate prices, cognizant that follower firms might be reticent to match the increase. Such a scenario can render the learning process from past coordination efforts more financially burdensome. This observation aligns with the proposition that frequent interaction fosters collusion (Calvano et al., 2020).

Divestiture impacts collusion incentives through a coordination effect, as collusion becomes more challenging with an increased number of firms due to the difficulty of building trust: λ has to be 1 to fully build the trust. The λ function characterizes the convergence of individual firms' beliefs, which converge to full awareness of collusion as historical information accumulates. As the number of firms grows, achieving collusion necessitates the convergence of all firms' beliefs towards rational expectations, an outcome that becomes increasingly difficult to reach.[21]

Table 5 presents the TBE model's prediction of a substantial decline in Salcobrand's likelihood to lead if a divestiture policy is implemented—approximately 67% for adjustment friction and 90% for divestiture. Conversely, the MPE model's predictions do not exhibit as significant a decrease—around 40% for adjustment friction and 30% for divestiture. This disparity suggests that the MPE model overlooks the coordination effect channel. The TBE model also proposes that divestiture is a more effective policy compared to adjustment friction, as it more significantly diminishes incentives for the firms. This decrease is relatively uniform across different stages of the trust-building process. The impact of adjustment friction decreases more initially. However, as trust builds up, the impact of adjustment friction does not match that of divestiture. In summary, the omission of the coordination effect during the initial stages of cartel formation by the MPE model leads to incorrect implications regarding the two policies. While the MPE model may suggest that adjustment friction is the more desirable policy, taking into account the trust-building process indicates that divestiture proves to be the more effective policy.

6 Conclusion

This paper broadens the understanding of firms' pricing decisions as they transition from non-collusive to collusive pricing strategies across various markets. A novel structural model is developed that incorporates an equilibrium concept, thereby facilitating exploration of firms' coordination with higher-order beliefs. A significant departure from previous research is the relaxation of the assumption

[21]The counterfactual paths are simulated using the following steps. First, I solve the model equilibrium CCPs for each counterfactual scenario for all observable state variables (x, h). Second, I simulate the price paths employing the equilibrium CCPs. For the adjustment friction experiment, I assume that the new high price is the minimum of the price following a 10% increase based on the low price and the original high price, while adjustment costs remain unchanged. For the divestiture experiment, I adjust the firms' payoffs as follows: (1) I assume that each of the three existing firms' scale of operation is reduced by 25%. (2) I assume that the new firm's menu cost and leadership cost coefficients are the average of those coefficients across the three existing firms. (3) I assume that the new firm's demand fixed effect is the average of the demand fixed effects of existing firms.

Table 5: Mean Predicted Leader Probability in Counterfactual Experiment - Salcobrand

	Trust Building Equilibrium			Markov Perfect Equilibrium		
	Model	Adjustment Friction	Divestiture	Model	Adjustment Friction	Divestiture
[0-30]	0.0316	0.0025 (-92.1619 %)	0.0029 (-90.9450%)	0.6096	0.3824 (-37.2706%)	0.4488 (-26.3775%)
[30-90]	0.0375	0.0048 (-87.3246%)	0.0030 (-92.1306%)	0.5887	0.2811 (-52.2456%)	0.4189 (-28.8447%)
[90-150]	0.1101	0.0132 (-88.0339%)	0.0055 (-94.9968%)	0.6092	0.3343 (-45.1228%)	0.4339 (-28.7686%)
[150+]	0.4725	0.1967 (-58.3724%)	0.0562 (-88.1068%)	0.6742	0.4953 (-26.5379%)	0.4462 (-33.8228%)
[All]	0.1629	0.0543 (-66.6871%)	0.0169 (-89.6400%)	0.6204	0.3733 (-39.8349%)	0.4370 (-29.5723%)

[1] This table presents the average probabilities of Salcobrand initiating price increases, encompassing 204 products.
[2] The second line details the percentage decrease in comparison to the model's prediction.

that data originates from an MPE, which assumes rational expectations of other firms' choice probabilities[22].

The model introduces a function for biased belief to account for firms' "learning-to-coordinate" behaviour, hypothesizing that this function eventually aligns with rational belief equilibria. Notably, the model provides an improved data fit and a superior explanation of firms' coordination compared to the standard MPE. This research provides new insights by modelling firms' flexible beliefs through a structure that captures all biases in beliefs. The biased belief function is identified using an extension of the methodology in Aguirregabiria and Magesan (2020), the key distinction being introducing a "signal" state that does not impact any player's payoff function. This "signal" state can serve as an equilibrium coordination device, endogenously determining the learning-to-coordinate process.

While other learning models such as fictitious play, Bayesian learning, and adaptive learning are effective, the learning process modelled in this paper provides a unique perspective, focusing on the transition from non-collusive to collu-

[22]As discussed in Green et al. (2014), Section 4, the assumptions of common knowledge and collusive equilibria do not withstand rigorous game-theoretic analysis, justifying the need to ease the rational expectation assumption when modelling the initiation stage of collusion.

sive equilibria. It is particularly beneficial when the two candidate equilibria are known and has potential applicability to scenarios involving multiple plausible collusive equilibria. However, this model also has limitations, requiring additional exclusion restrictions for identification.

This research also contributes to policy discussions by evaluating two potential interventions through counterfactual experiments. The first intervention considers the increase of price adjustment cost, while the second follows the structural remedy suggested by Harrington Jr (2018), which involves divesting firms to form a new competitor. The findings suggest that both policies can prevent colluding firms from reaching a subgame perfect equilibrium. However, the implications for policy are nuanced. Without considering the trust-building process, the effect of the divestiture policy could be underestimated.

A Proofs

A.1 Lemmas and Proofs

Lemma 1. *Under assumptions 1, 2, 3, and 4, with known $\lambda(h)$ and $\mathbf{P}$, the structural payoffs $\pi_{im}(\mathbf{a}_{im}, x_{im}^{(1)}) - \pi_{im}(\mathbf{a}_{im}, x_{im}^{(2)})$ and any two points $x_{im}^{(1)}, x_{im}^{(2)} \in \mathcal{X}_i$.*

Proof of Lemma 1. Without loss of generality, I focus on the identification for firm i. Note that with a known $\lambda(h)$, for any given history state h, I can compute the belief function $\mathbf{B}_{im}^{h}$ based on Assumption 2. Assumptions 2 and 3 correspond to Assumptions ID-1 and ID-2 of Aguirregabiria and Magesan (2020) (AM19 in the rest of the proof). Assumption 4(A)-(C) corresponds to Assumptions ID-3 (i), (iii), and (iv) of AM19. Assumption ID-4 (ii) follows naturally with the definition of belief stated in Assumption 2. Assumption 1 corresponds to AM19 Assumptions MOD-1 to MOD-3, and AM19 Assumption MOD-4 is a natural implication of AM19 Assumption MOD-1. AM19 Assumption MOD-5 is satisfied naturally in my model by the definition of the discretized state variable.

I assume that the joint distribution of x_{im}, x_{-im} over the population of M markets where I observe these variables has a strictly positive probability at every point in the joint support set $\mathcal{X}_m$. With assumption 3, the identification of the vector of value differences, $\tilde{\mathbf{v}}_{im}^{\mathbf{B}_i^h}(\mathbf{x})$, follows Hotz and Miller (1993) (*Proposition 3*), where

$\tilde{\mathbf{v}}_{im}^{\mathbf{B}_i^h}(\mathbf{x}) = \{v_{im}^{\mathbf{B}_i^{h_t}}(a_{im}, \mathbf{x}), a_{im} \in \mathcal{A}\}$ is a vector of choice-specific value differences as defined in equation (6). With assumption 4, I write $\tilde{\mathbf{v}}_{im}^{\mathbf{B}_i^h}(\mathbf{x})$ as

$$v_{im}^{\mathbf{B}_i^{h_t}}(a_i, \mathbf{x}) = \Big(\mathbf{B}_{im}^h(\cdot, \mathbf{x})\Big)^\top \tilde{\mathbf{g}}_{im}^{\mathbf{B}_i^h}(a_{im}, \mathbf{x}), \tag{13}$$

where $\mathbf{B}_{im}^h(\cdot, \mathbf{x}) = \{\mathbf{B}_{im}^h(\mathbf{a}_{-im}, \mathbf{x}) : \mathbf{a}_{-im} \in \mathcal{A}_{-im}\}$ and $\tilde{\mathbf{g}}_{im}^{\mathbf{B}_i^h}(a_{im}, \mathbf{x}) = \{\tilde{\mathrm{g}}_{im}^{\mathbf{B}_i^h}(a_{im}, a_{-im}, \mathbf{x}) : \mathbf{a}_{-im} \in \mathcal{A}_{-im}\}$ are vectors that contains the beliefs and $\tilde{\mathrm{g}}_{im}$-function for every possible value in the action space conditioned on the state. The $\tilde{\mathrm{g}}_{im}$-function takes the form

$$\begin{aligned}
&\tilde{\mathrm{g}}_{im}^{\mathbf{B}_i^{h_t}}(a_{im}, a_{-im}, \mathbf{x}) = \tilde{\pi}_{im}(a_{im}, a_{-im}, x_{im}) \\
&\quad + \beta \sum_{\substack{x_{m,t+1} \\ \in \mathcal{X}_{im}}} \tilde{f}_x\big(\mathbf{x}_{m,t+1} | (a_{im}, a_{-im})\big) \sum_{\substack{h_{t+1} \\ \in \mathcal{H}}} f_{h,im}^{\mathbf{B}_i^{h_t}, \mathbf{P}_i^{h_t}}\big(h_{t+1} | a_{im}, \mathbf{a}_{-im}, h_t\big) \bar{V}_{im}^{\mathbf{B}_{im}^h}(\mathbf{x}_{t+1}),
\end{aligned} \tag{14}$$

where $\tilde{\pi}_{im}(a_{im}, a_{-im}, x_{im}) = \pi_{im}(a_{im}, a_{-im}, x_{im}) - \pi_{im}(0, a_{-im}, x_{im})$ is the flow payoff difference. $\tilde{f}_x\big(\mathbf{x}_{im,t+1} | (a_{im}, a_{-im})\big) = f_x\big(\mathbf{x}_{m,t+1} | (a_{im}, a_{-im})\big) - f_x\big(\mathbf{x}_{m,t+1} | (0, a_{-im})\big)$ is the transition density difference. $f_{h,im}^{\mathbf{B}_i^{h_t}, \mathbf{P}_i^{h_t}}\big(h_{t+1} | a_{im}, \mathbf{a}_{-im}, h_t\big)$ is the expected transition density of h_t given that the action on market m is $(a_{im}, \mathbf{a}_{-im})$ and firm i behave according to $\mathbf{P}_i^{h_t}$ and believe other firms will behave according to $\mathbf{B}_i^{h_t}$ on other markets m':

$$\begin{aligned}
&f_{h,im}^{\mathbf{B}_i^{h_t}, \mathbf{P}_i^{h_t}}\big(h_{t+1} | a_{im}, \mathbf{a}_{-im}, h_t\big) = \\
&\sum_{\substack{\mathbf{a}_i = (a_{im}, \{a_{im'}, m' \in \mathcal{M}\}), \\ \mathbf{a}_{-i} = (\mathbf{a}_{-im}, \{\mathbf{a}_{-im'}, m' \in \mathcal{M}\})}} \Pi_{m' \in \mathcal{M}, m' \neq m} \mathbf{B}_{im'}^{h_t}(\mathbf{a}_{-im'}, \mathbf{x}) \mathbf{P}_{im'}^{h_t}(a_{im'}, \mathbf{x}) f_h(h_{t+1} | \mathbf{a}_i, \mathbf{a}_{-i}, h_t).
\end{aligned}$$

The first part of AM19 Proposition 2(2.2)-(2.3) shows that with Assumptions 1, 2, 3, 4 and with known beliefs, $\tilde{\mathrm{g}}_{im}^{\mathbf{B}_i^{h_t}}(a_{im}, a_{-im}, \mathbf{x})$ is identified everywhere for $(a_{im}, \mathbf{a}_{-im}, \mathbf{x}) \in \mathcal{A}_i \times \mathcal{A}_{-i} \times \mathcal{X}$.

Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be two points in the state space such that all other elements are the same except for firm i's lagged pricing decision on market m: $\mathbf{x}^{(1)} = \{\mathbf{x}_{im}^{(1)}, \mathbf{x}_{-im}, \{\mathbf{x}_{m'}, m' \neq m, m' \in \mathcal{M}\}$ and $\mathbf{x}^{(2)} = \{\mathbf{x}_{im}^{(2)}, \mathbf{x}_{-im}, \{\mathbf{x}_{m'}, m' \neq m, m' \in \mathcal{M}\}$. I have $\tilde{\mathrm{g}}_{im}^{\mathbf{B}_i^{h_t}}(\mathbf{a}_{im}, \mathbf{a}_{-im}, \mathbf{x}^{(1)}) - \tilde{\mathrm{g}}_{im}^{\mathbf{B}_i^{h_t}}(\mathbf{a}_{im}, \mathbf{a}_{-im}, \mathbf{x}^{(2)}) = \tilde{\pi}_{im}(\mathbf{a}_{im}, \mathbf{x}_{im}^{(1)}) - \tilde{\pi}_{im}(\mathbf{a}_{im}, \mathbf{x}_{im}^{(2)})$ identified everywhere for $(\mathbf{a}_{im}, \mathbf{x}_m) \in \mathcal{A}_i \times \mathcal{X}_m$ and for all markets $m \in \mathcal{M}$. $\square$

A.2 Proof of Propositions

Proof of Proposition 1. Lemma 1 shows that if the biased belief functions $\lambda(h)$ and $\mathbf{P}_{it}$ are known, the structural payoffs are identified everywhere. Let $x_{im}^{(1)} = 1, x_{im}^{(2)} = 0$, therefore

$$\pi_{im}(\mathbf{a}_m, x_{im}^{(1)}) - \pi_{im}(\mathbf{a}_m, x_{im}^{(2)}) = \Big(\mathbf{R}_{im}(\mathbf{a}_m) + \mathbf{A}_{im}(\mathbf{a}_m, x_{im}^{(1)})\Big) - \Big(\mathbf{R}_{im}(\mathbf{a}_m) + \mathbf{A}_{im}(\mathbf{a}_m, x_{im}^{(2)})\Big),$$

where $\mathbf{R}_{im}$ and F_{im} are defined in (2) and (4).

Because $\mathbf{R}_{im}$ only depends on $\mathbf{a}_m$, the difference $\pi_{im}(\mathbf{a}_m, x_{im}^{(1)}) - \pi_{im}(\mathbf{a}_m, x_{im}^{(2)})$ assumes the form

$$\mathbf{A}_{im}(\mathbf{a}_m, x_{im}^{(1)}) - \mathbf{A}_{im}(\mathbf{a}_m, x_{im}^{(2)}) = \Big(\mathrm{MC}_{im}\mathbb{1}(a_{im} \neq 1)\Big) - \Big(\mathrm{MC}_{im}\mathbb{1}(a_{im} \neq 0) + a_{im}\mathbb{1}(\mathbf{a}_{-im} = \mathbf{0})\mathrm{LC}_{im}\Big)$$

and is identified for every $(a_{im}, \mathbf{a}_{-im}) \in \mathcal{A}_i \times \mathcal{A}_{-i}$; thus, MC_{im} and LC_{im} have been identified. The identification of $\boldsymbol{\theta}_{i2}$ follows naturally.

$\square$

Proof of Proposition 2. Under assumptions 1, 2, 3, and 4, I have shown that the value function $v_{im}^{\mathbf{B}_i^{h_t}}(a_{im}, \mathbf{x})$ are identified and takes the representation as in (13). Let $\mathcal{X}^{(1)} \subset \mathcal{X}$ be a subset of the state space that satisfies the condition that $\mathcal{X}^{(1)} \equiv \{\mathbf{x} = (\mathbf{x}_{im}^{(1)}, \mathbf{x}_{-im}, \{\mathbf{x}_{m'} : m' \neq m, m' \in \mathcal{M}\}), \mathbf{x} \in \mathcal{X}\}$ for $x_{im}^{(1)} \in \mathcal{X}$. Then given the exclusion restriction as in Assumption 4, for any $\mathbf{x}_{mt} \in \mathcal{X}^{(1)}$, I have representation $v_{im}^{\mathbf{B}_i^{h_t}}(a_{im}, \mathbf{x}) = \Big(\mathbf{B}_{im}^{h}(\cdot, \mathbf{x})\Big)^\top \tilde{\mathbf{g}}_{im}^{\mathbf{B}_i^{h}}(a_{im}, \mathbf{x})$. By Assumption 2, $\mathbf{B}_{im}^{h}(\cdot, \mathbf{x}) = \Xi(\lambda(h), \mathbf{P}_{-i}^{h}(\cdot, \mathbf{x}))$. Stack all conditions over $\mathcal{X}^{(1)}$, I have

$$\underbrace{\mathbf{Q}_{im}^{h,(1)}(a_{im})}_{|\mathcal{X}^{(1)}|\times 1} = \underbrace{\mathbf{P}_{-im}^{h,(1)}}_{|\mathcal{X}^{(1)}|\times|\mathcal{A}_{-i}|} \underbrace{(\mathrm{M}^{\lambda(h)})^\top \tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}^{h}}(a_{im}, \mathbf{x})}_{|\mathcal{A}_{-i}|\times 1}, \tag{15}$$

where $\mathbf{Q}_{im}^{h,(1)}(a_{im})$ is a column vector, with elements $v_{im}^{\mathbf{B}_i^{h_t}}(a_{im}, \mathbf{x})$ for $\mathbf{x} \in \mathcal{X}^{(1)}$; and $\mathbf{P}_{-im}^{h,(1)}$ is a matrix with row vectors $\big(\mathbf{P}_{-im}^{h}(\cdot, \mathbf{x})\big)^\top$ for $\mathbf{x} \in \mathcal{X}^{(1)}$.

Because $\mathbf{Q}_{im}^{h,(1)}(a_{im})$ and $\mathbf{P}_{-im}^{h,(1)}$ are identified, with the condition $|\mathcal{X}^{(1)}| \geq |\mathcal{A}_{-im}|$ naturally holds, then

$$\boldsymbol{\delta}_{im}^{h,(1)} \equiv (\mathrm{M}^{\lambda(h)})^\top \tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}^{h}}(a_{im}, \mathbf{x}) = \Big((\mathbf{P}_{-im}^{h,(1)})^\top \mathbf{P}_{-im}^{h,(1)}\Big)^{-1} (\mathbf{P}_{-im}^{h,(1)})^\top \mathbf{Q}_{im}^{h,(1)}(a_{im}),$$

and $\boldsymbol{\delta}_{im}^{h,(1)}$ is identified for each pair of $(h, x_{im}^{(1)}) \in \mathcal{H} \times \mathcal{X}_i$. $\square$

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